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Lie Triple Hom Γ-derivation and Jordan Triple Hom Γ-derivation on Bi-Hom Γ-Lie algebra

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Abstract
The purpose of this paper is to define Bi- Hom Γ-Lie algebra, Lie triple Hom Γ-derivation on Bi- Hom Γ-Lie algebra $(g,[.,.]_\lambda,\theta_1,\theta_2)$and Jordan triple Hom Γ-derivation on Bi-Hom Γ-Lie algebra $(g,[.,.]_\lambda,\theta_1,\theta_2)$, Also to study the relation between Lie triple Hom Γ-derivation and Jordan triple Hom Γ-derivation on Bi-Hom Γ-Lie algebra $(g,[.,.]_\lambda,\theta_1,\theta_2)$.

Keywords: Derivation, Lie algebra , Jordan Derivation , Lie Derivation

اشتقاق كاما هوم لي الثلاثي واشتقاق كاما هوم جوردان الثلاثي على كاما هوم جبر لي الثنائي

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الخلاصة
الغرض من هذا البحث هو تعريف كاما هوم جبر لي الثنائي، اشتقاق كاما هوم لي الثلاثي على كاما هوم جبر لي الثنائي$(g,[.,.]_\lambda,\theta_1,\theta_2)$ و اشتقاق كاما هوم جوردان الثلاثي على كاما هوم جبر لي الثنائي$(g,[.,.]_\lambda,\theta_1,\theta_2)$، ايضا لدراسة العلاقة بين اشتقاق كاما هوم لي الثلاثي و اشتقاق كاما هوم جوردان الثلاثي على كاما هوم جبر لي الثنائي $(g,[.,.]_\lambda,\theta_1,\theta_2)$

1. Introduction

It is indispensable that derivation and generalized derivation algebras are very substantial topic in the study of Lie algebras. In [1] Leger and Luks investigated the construction of the generalized derivation of Lie algebras. In [2] Benoist Generalized derivations also have a crucial role in Benoist study of Levi principles in derivation algebras of nilpotent Lie algebras .In [3] Bresar and vukman generalized Herstein result to Jordan (α , β) – derivation. In [4] Ashraf and Ali proved that in non-commutative prime ring R, a generalized Jordan derivation is a generalized (α, β)- derivation when α is an automorphism of R. Nowadays in [5,9], there is an increasing interest in studying the Lie triple derivation. In [10] Sheng, the Hom Lie algebra is a reasonable generalization of the Lie algebra and has significant applications in both maths. and phys. In [11] Bing and Chen identify the generalized derivations for Hom Lie algebras. In short, Lie triple (α, β)-Hom derivations and Jordan triple (α, β) -Hom derivations on Hom Lie algebra are studied .In [12] Rezaei and Davvaz , generalize the aspect of algebra over a field. A Γ- algebra is an algebraic structure consisting of a vector space V, a groupoid Γ together with a map from V× Γ × V to V. Then, on every associative Γ-algebra V and for

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every $\alpha \in \Gamma$ establish an α-Lie algebra.In this study we generalize the above results to Bi-Hom Γ-Lie algebra,we proved that Jordan triple (α, β) Hom Γ-derivation if and only if Lie triple (α, β)-Hom Γ-derivations on Bi-Hom Γ-Lie algebra under some conditions. In particular , α-Jordan Hom Γ-derivation if and only if Lie triple α-Hom Γ-derivation on Bi-Hom Γ-Lie algebra. For more result on $\Gamma -$Lie algebra, see [13,14] Throughout this paper, the main field F is presupposed to be of features not equal to 3. We now recall some primary definitions. In [15] Zhou, construct a Lie triple algebra L_T by defining a triple product $[r,p,w] = \big[r,[p,w]\big]$, Where $[r,p]$ is the brocket of elements r,p in the Lie algebra.
Now, we will recall the followings which are necessary in this paper.

Definition 1.1: [16]

A Hom Lie algebra is a triple $(A,[.,.],\theta)$ consisting of a vector space , a skew symmetric bilinear map $[.,.]: A \times A \longrightarrow A$, and a linear map θ: $A \longrightarrow A$ satisfying the following hom - Jacobi identity$\big[\theta(r),[p,w]\big] + \big[\theta(p),[w,r]\big] + \big[\theta(w),[r,p]\big] = 0$

Definition 1.2:- [17]

A Bi-Hom Lie algebra over a field K is a 4- tuple $(L,[.,.],\theta_1,\theta_2)$, where L is a K-linear spase $\theta_1: L \to L$, $\theta_2: L \to L$ and$[\,,]: L \times L \to$L are linear maps, with notation $[\,,](\mathfrak{r},\mathfrak{p}) = [\mathfrak{r},p]$, satisfying the following conditions, for all $r,p,w \in L$

1- $\theta_1 \circ \theta_2 = \theta_2 \circ \theta_1$

2- $[\theta_1(r),\theta_2(p)] = -[\theta_1(p),\theta_2(r)]$

3-$\big[{\theta_2}^2(r),[\theta_2(p),\theta_1(w)]\big] + \big[{\theta_2}^2(p),[\theta_2(w),\theta_1(r)]\big] + \big[{\theta_2}^2(w),[\theta_2(r),\theta_1(p)]\big]$= 0.

Definition 1.3: [11]

A Lie triple derivation of Lie algebra is a linear mapping $D: A \longrightarrow A$ such that $D\big([[r,p],w]\big) = \big[[D(r),p],w\big] + \big[[r,D(p)],w\big] + \big[[r,p],D(w)\big]$,for all $r,p,w \in A$.

Definition 1. 4: [11]

A Jordan triple derivation of a Lie algebra is a linear mapping $\acute{D}: A \longrightarrow A$ such that $\acute{D}\big([[r,p],r]\big) = \big[[\acute{D}(r),p],r\big] + \big[[r,\acute{D}(p)],r\big] + \big[[r,p],\acute{D}(r)\big]$, for all $r,p \in A$.

2- Main Results

In this section, we give a definition of Bi-Hom Γ – Lie algebra , Lie triple Hom Γ-derivation on Bi-Hom Γ-Lie algebra and Jordan triple Hom Γ- derivation on Bi-Hom Γ-Lie algebra. We will use the notation L for Lie, L.T for Lie triple, J.T for Jordan triple.

Definition 2.1:

A Bi-Hom Γ-(L) algebra over a field K is a 4 - tuple $(L,[.,.]_\lambda,\theta_1,\theta_2)$, where L is a K-linear spase $\theta_1: L \to L$, $\theta_2: L \to L$ and $[\,,]_\lambda: L \times L \to L$ are linear maps, with notation $[\,,]_\lambda(r,p) = [r,p]_\lambda$, satisfying the following conditions, for all $r,p,w \in$ L, $\lambda \in \Gamma$.

1- $\theta_1 \circ \theta_2 = \theta_2 \circ \theta_1$

2- $[\theta_1(r),\theta_2(p)]_\lambda = -[\theta_1(p),\theta_2(r)]_\lambda$

3- $\big[{\theta_2}^2(r),[\theta_2(p),\theta_1(w)]_\lambda\big]_\lambda + \big[{\theta_2}^2(p),[\theta_2(w),\theta_1(r)]_\lambda\big]_\lambda + \big[{\theta_2}^2(w),[\theta_2(r),\theta_1(p)]_\lambda\big]_\lambda =$ 0.

Definition 2.2:
A L. T Hom Γ-derivation on Bi-Hom Γ- L algebra $(A,[.,.]_\lambda,\theta_1,\ \theta_2)$ is a Linear map $D: A \longrightarrow A$ such that $D\circ\theta_1=\theta_1\circ D, D\circ\theta_2=\theta_2\circ D$ and $\delta: A\to A$ is a homomorphism such that $\delta\circ\theta_1=\ \theta_1\circ\delta$, $\delta\circ\theta_2=\theta_2\circ\delta$,
$D([[r,p]_\lambda,w]_\lambda)=\left[\left[D(r),{\theta_1}^k\ (p)\right]_\lambda,{\theta_2}^k(w)\right]_\lambda+\left[\left[{\theta_1}^k(r),D(p)\right]_\lambda,{\theta_2}^k(w)\right]_\lambda+$
$\left[\left[{\theta_1}^k(r),{\theta_2}^k(p)\right]_\lambda,D(w)\right]_\lambda+\delta[[r,p]_\lambda,w]_\lambda$,for all r, $p,w\in A,\lambda\in\Gamma$.

Definition 2.3:
A J. T Hom Γ-derivation on Bi-Hom Γ- L algebra $(A,[.,.]_\lambda,\theta_1,\theta_2)$ is a Linear map $\acute{D}: A\longrightarrow A$ such that $\acute{D}\circ\theta_1=\theta_1\circ\acute{D}$, $\acute{D}\circ\ \theta_2=\theta_2\circ\acute{D}$ and $\delta: A\to A$ is a homomorphism such that $\delta\circ\theta_1=\ \theta_1\circ\delta$, $\delta\circ\theta_2=\theta_2\circ\delta$,
$\acute{D}([[r,p]_\lambda,r]_\lambda)=\left[\left[\acute{D}(r),{\theta_1}^k(p)\right]_\lambda,{\theta_2}^k(r)\right]_\lambda+\left[\left[{\theta_1}^k(r),\acute{D}\ (p)\right]_\lambda,{\theta_2}^k(r)\right]_\lambda$
$+\ \left[\left[{\theta_1}^k(r),{\theta_2}^k(p)\right]_\lambda,\acute{D}(r)\right]_\lambda+\delta[[r,p]_\lambda,r]_\lambda$, for all $r,p\in A,\lambda\in\Gamma$.

Definition 2.4:
Let $(A,[.,.]_\lambda,\theta_1,\theta_2)$ be a Bi-Hom Γ- L algebra and let D $,\alpha,\beta: A\longrightarrow A$ be Linear maps satisfying $D\circ\theta_1=\theta_1\circ D, D\circ\theta_2=\theta_2\circ D$ and $\delta: A\to A$ is a homomorphism such that $\delta\circ\theta_1=\ \theta_1\circ\delta$, $\delta\circ\theta_2=\theta_2\circ\delta$,
1- D is called a L. T $(\alpha,\beta)_1$- Hom Γ-derivation on Bi-Hom Γ- L algebra if
$D([[r,p]_\lambda,w]_\lambda)=\left[\left[D(r),\alpha({\theta_1}^k\ (p))\right]_\lambda,\beta\left({\theta_2}^k(w)\right)\right]_\lambda+$
$\left[\left[\alpha\left({\theta_1}^k(r)\right),D(p)\right]_\lambda,\beta\left({\theta_2}^k\ (w)\right)\right]_\lambda$ $+\left[\left[\alpha\left({\theta_1}^k(r)\right),\beta\left({\theta_2}^k(p)\right)\right]_\lambda,D(w)\right]_\lambda+$
$\delta[[r,p]_\lambda,w]_\lambda$.
2- D is called a L. T $(\alpha,\beta)_2$- Hom Γ-derivation on Bi-Hom Γ- L algebra if
$D([[r,p]_\lambda,w]_\lambda)=\left[\left[D(r),\alpha\left({\theta_1}^k(p)\right)\right]_\lambda,\alpha\left({\theta_2}^k(w)\right)\right]_\lambda+$
$\left[\left[\alpha\left({\theta_1}^k(r)\right),D(p)\right]_\lambda,\beta({\theta_2}^k(w)\ )\right]_\lambda+\left[\left[\beta({\theta_1}^k(r),\beta({\theta_2}^k(p))\right]_\lambda,D(w)\right]_\lambda$ $+\delta[[r,p]_\lambda,w]_\lambda$.
3- D is called a L. T $(\alpha,\beta)_3$- Hom Γ-derivation on Bi-Hom Γ- L algebra if
$D([[r,p]_\lambda,w]_\lambda)=\left[\left[D(r),\alpha\left({\theta_1}^k(p)\right)\right]_\lambda,\alpha({\theta_2}^k(w)\right]_\lambda$
$+\left[\left[\beta\left({\theta_1}^k(r)\right),D(p)\right]_\lambda,\alpha({\theta_2}^k(w))\right]_\lambda+\left[\left[\beta\left({\theta_1}^k(r)\right),\beta\left({\theta_2}^k(p)\right)\right]_\lambda,D(w)\right]_\lambda+$
$\delta[[r,p]_\lambda,w]_\lambda$,for all r , p , $w\in A,\lambda\in\Gamma$.
If $w=r$ we can defined a J. T $(\alpha,\beta)_2$-Hom Γ-derivation on Bi-Hom Γ- L algebra by the same way it is easy to see that if D is a L. T (α,β)i- Hom Γ-derivati of $(A,[.,.]_\lambda,\theta_1,\theta_2)$ where is, i = 1, 2, 3.In this section, $(A,\ [.,.]_\lambda,\theta_1,\theta_2)$ is Bi-Hom Γ- L algebra and $\alpha,\beta: A\longrightarrow A$ be a Linear mappings, where θ_i is an injection and $\theta_i[r,p]_\lambda=[r,\theta_i(p)]_\lambda,\theta_2$ be a homomorphism

Definition 2.5:
Let $(A,[.,.]_\lambda,\theta_1,\theta_2)$ be a Bi-Hom Γ- (L) algebra and let $D,\alpha,\beta: A\longrightarrow A$ be linear map satisfying, $D\circ\theta_1=\theta_1\circ D, D\circ\theta_2=\theta_2\circ D$ and $\delta: A\to A$ is a homomorphism such that $\delta\circ\theta_1=\ \theta_1\circ\delta$, $\delta\circ\theta_2=\theta_2\circ\delta$,
1- D is called a J. T $(\alpha,\beta)_1$ −Hom Γ-derivation on Bi-Hom Γ- L algebra if
$D([[r,p]_\lambda,r]_\lambda)=\left[\left[D(r),\alpha\left({\theta_1}^k(p)\right)\right]_\lambda,\beta\left({\theta_2}^k(r)\right)\right]_\lambda$

$+\left[\left[\alpha\left(\theta_1{}^k(r)\right),D(p)\right]_\lambda,\beta\left(\theta_2{}^k(r)\right)\right]_\lambda$
$+\left[\left[\alpha\left(\theta_1{}^k(r)\right),\beta\left(\theta_2{}^k(p)\right)\right]_\lambda,D(r)\right]_\lambda+\delta[[r,p]_\lambda,r]_\lambda,\forall\, r,p\in A.$

2- D is called **a** J. T $(\alpha,\beta)_2$-HomΓ-derivation on Bi-Hom Γ- L algebra if
$D([[r,p]_\lambda,r]_\lambda)=\left[\left[D(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,\alpha\left(\theta_2{}^k(r)\right)\right]_\lambda$
$=\left[\left[D(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,\alpha\left(\theta_2{}^k(r)\right)\right]_\lambda+\left[\left[\alpha\left(\theta_1{}^k(r)\right),D(p)\right]_\lambda,\beta\left(\theta_2{}^k(r)\right)\right]_\lambda$
$+\left[\left[\beta\left(\theta_1{}^k(r)\right),\beta\left(\theta_2{}^k(p)\right)\right]_\lambda,D(r)\right]_\lambda+\delta[[r,p]_\lambda,r]_\lambda,\forall\, r,p\in A.$

3- D is called a J. T $(\alpha,\beta)_3$-Hom Γ-derivation on Bi-Hom Γ- L algebra
$D([[r,p]_\lambda,r]_\lambda)=\left[\left[D(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,\alpha\left(\theta_2{}^k(r)\right)\right]_\lambda+$
$\left[\left[\beta\left(\theta_1{}^k(r)\right),D(p)\right]_\lambda,\alpha\left(\theta_2{}^k(r)\right)\right]_\lambda$
$+\left[\left[\beta\left(\theta_1{}^k(r)\right),\beta\left(\theta_2{}^k(p)\right)\right]_\lambda,D(r)\right]_\lambda+\delta[[r,p]_\lambda,r]_\lambda$, for all $r,p,\in A,\lambda\in\Gamma$

Theorem 2.6:

D is a L. T $(\alpha,\beta)_1$ – Hom Γ-derivation if and only if D is a J. T $(\alpha,\beta)_1$ Hom Γ-derivation on Bi-Hom Γ- L algebra $(A,[.,.]_\lambda,\theta_1,\theta_2)$ such that:

1- $\left[\left[\alpha\left(\theta_1{}^k(r)\right),\beta\left(\theta_2{}^k(p)\right)\right]_\lambda,D(w)\right]_\lambda=\left[\left[\beta\left(\theta_2{}^k(r)\right),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,D(w)\right]_\lambda.$

2- $B(r,p,w)+B(p,w,r)+B(w,r,p)=0$, where $r,p,w\in$ A and
$B(r,p,w)=\theta_2o\theta_1\left(\left[\left[D(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda+\right.$
$\left.\left[\left[\alpha\left(\theta_1{}^k(r),D(p)\right)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda+\left[\left[\alpha\left(\theta_1{}^k(r)\right),\beta\left(\theta_2{}^k(p)\right)\right]_\lambda,D(w)\right]_\lambda\right)$
$+\delta[[r,p]_\lambda,w]_\lambda$. And θ_2 be a homomorphism $\theta_1\,[r\,,p]_\lambda=\,[r\,,\theta_1(p)]_\lambda$ and ch A $\neq 3$

Proof:

Assume that D is a (L. T) $(\alpha,\beta)_1$ - a Hom Γ-derivation of $(A,[.,.]_\lambda,\theta_1,\theta_2)$. Clearly D is a (J. T) $(\alpha,\beta)_1$- Hom Γ-derivation of $(A,[.,.]_\lambda,\theta_1,\theta_2)$ and
$D\left([[[r,p]_\lambda],w]_\lambda\right)=-D\,([[p,r]_\lambda,w]_\lambda)=$
$-\left[\left[D(p),\alpha\left(\theta_1{}^k(r)\right)\right]_\lambda,\beta(\,\theta_2{}^k(w))\right]_\lambda-\left[\left[\alpha\left(\theta_1{}^k(p)\right),D(r)\right]_\lambda,\beta\left(\theta_2{}^k(p)\right)\right]_\lambda$
$-\left[\left[\alpha\left(\,\theta_1{}^k(p)\right),\beta\left(\theta_2{}^k(r)\right)\right]_\lambda,D(w)\right]_\lambda-\delta[[r,p]_\lambda,w]_\lambda$
$=\left[\left[D\,(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda+\left[\left[D\,(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda$
$+\left[\left[\beta\left(\theta_2{}^k(r)\right),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,D(w)\right]_\lambda+\delta[[r,p]_\lambda,w]_\lambda$. And
$D([[r,p]_\lambda,w]_\lambda)=\left[\left[D(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda$
$+\left[\left[\alpha\left(\theta_1{}^k(r)\right),D(p)\right]_\lambda,\beta(\theta_2{}^k(w))\right]_\lambda+\left[\left[\alpha\left(\theta_1{}^k(r)\right),\beta\left(\theta_2{}^k(p)\right)\right]_\lambda,D\,(w)\right]_\lambda+$
$\delta[[r,p]_\lambda,w]_\lambda$, Then we get 1
$\left[\left[\beta\left(\theta_2{}^k(r)\right),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,D(w)\right]_\lambda=\left[\left[\alpha\left(\theta_1{}^k(r)\right),\beta\left(\theta_2{}^k(p)\right)\right]_\lambda,D\,(w)\right]_\lambda.$

Since D is a (L. T) $(\alpha,\beta)_1$-Hom Γ- derivation of $(A,[.,.]_\lambda,\theta_1,\theta_2)$, we have
$\theta_2o\theta_1\big(D([[r,p]_\lambda,w]_\lambda)\big)=B(r,p,w)$, Hence
$B(r,p,w)+B(p,w,r)+B(w,r,p)$

$= \theta_2 o\theta_1\big((D[[r,p]_\lambda,w]_\lambda) + D([[p,w]_\lambda,r]_\lambda) + D([[w,r]_\lambda,p]_\lambda)\big)$
$= D[[\theta_2(r),\theta_2(p)]_\lambda,\theta_1(w)]_\lambda + [[\theta_2(p),\theta_2(w)]_\lambda,\theta_1(r)]_\lambda + [[\theta_2(w),\theta_2(r)]_\lambda,\theta_1(p)]_\lambda = 0$
Conversely, let D be a J. T $(\alpha,\beta)_1$- Hom Γ- derivation of $(A,[.,.]_\lambda,\theta_1,\theta_2)$ for which 1 , 2 conditions hold then, $\theta_2 o\theta_1\big(D([[r,p]_\lambda,r]_\lambda)\big) = B(r,p,r)$. It follows that
$\theta_2 o\theta_1\big(D([[r+w,p]_\lambda,r+p]_\lambda)\big)$
$= \theta_2 o\theta_1\big(D([[r,p]_\lambda,r]_\lambda)\big) + D([[r,p]_\lambda,w]_\lambda) + D([[w,p]_\lambda,r]_\lambda) + D([[w,p]_\lambda,w]_\lambda)$
$= B(r,p,r) + B(w,p,w) + \theta_2 o\theta_1\big(D([[r,p]_\lambda,w]_\lambda)\big) + \theta_2 o\theta_1\big(D([[w,p]_\lambda,r]_\lambda)\big)$, And
$\theta_2 o\theta_1\big(D([[r+w,p]_\lambda,r+w]_\lambda)\big) = B(r+w,p,r+w)$
$= B(r,p,r) + B(r,p,w) + B(w,p,r) + B(w,p,w)$
thus we obtain
$\theta_2 o\theta_1(D([[r,p]_\lambda,w]_\lambda) + \theta_2 o\theta_1\big(D([[w,p]_\lambda,r]_\lambda)\big)$
$= B(r,p,w) + B(w,p,r)$ …………………….. (1)

$B(p,r,w) = \theta_2 o\theta_1\left(\left[\left[D(p),\alpha\left(\theta_1{}^k(r)\right)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(p),D(r)\right)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(p)\right),\beta\left(\theta_2{}^k(r)\right)\right]_\lambda,D(w)\right]_\lambda\right)$
$+\delta[[p,r]_\lambda,w]_\lambda$
$= -\theta_2 o\theta_1\left(\left[\left[\left[D(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda\right],\beta\left(\theta_2{}^k(w)\right)\right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(r)\right),D(p)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda + \left[\left[\beta\left(\theta_1{}^k(r)\right),\alpha\left(\theta_2{}^k(p)\right)\right]_\lambda,D(w)\right]_\lambda\right)$
$+\delta[[r,p]_\lambda,w]_\lambda$
$= -\theta_2 o\theta_1\left(\left[\left[D(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(r)\right),D(p)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(r)\right),\beta\left(\theta_2{}^k(p)\right)\right]_\lambda,D(w)\right]_\lambda\right) + \delta[[r,p]_\lambda,w]_\lambda$
$= -B(r,p,w) \Longrightarrow \theta_2 o\theta_1\big(D([[r,p]_\lambda,p]_\lambda)\big) = -\theta_2 o\theta_1\big(D([[p,r]_\lambda,p]_\lambda)\big)$
$= -B(p,r,p) = B(r,p,p)$. A similar argument proves
$\theta_2 o\theta_1\big(D([[r,p]_\lambda,w]_\lambda)\big) + \theta\big(D([[r,w]_\lambda,p]_\lambda)\big)$
$= B(r,p,w) + B(r,w,p)$ …………………. (2)

By (1) and (2), we have
$\theta_2 o\theta_1(D[[r,p]_\lambda,w]_\lambda) + \theta_2 o\theta_1(D[[w,p]_\lambda,r]_\lambda) + \theta_2 o\theta_1(D[[r,p]_\lambda,w]_\lambda) + \theta_2 o\theta_1(D[[r,w]_\lambda,p]_\lambda)$
$= B(r,p,w) + B(w,p,r) + B(r,p,w) + B(r,w,p)$. Then
$\theta_2 o\theta_1\big((D[[r,p]_\lambda,w]_\lambda)\big) + \theta_2 o\theta_1\big(D([[r,p]_\lambda,w]_\lambda)\big) - \theta_2 o\theta_1\big(D([[p,r]_\lambda,w]_\lambda)\big)$
$= B(r,p,w) + B(w,p,r) + B(r,p,w) + B(r,w,p)$. That is,
$3\theta_2 o\theta_1\big(D([[r,p]_\lambda,w]_\lambda)\big) = 3B(r,p,w)$
$+B(w,p,r) + B(r,w,p) + B(p,r,w) = 3B(r,p,w)$,
Where the last equation (ii) is used. Since $ch\ A \neq 3$, we have
$\theta_2 o\theta_1\big(D([[r,p]_\lambda,w]_\lambda)\big) = B(r,p,w)$, and so
$D([[r,p]_\lambda,w]_\lambda) =$
$\left[\left[D(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(r),\ D(p)\right)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda$
$+ \left[\left[\alpha\left(\theta_1{}^k(r)\right),\beta\left(\theta_2{}^k(p)\right)\right]_\lambda,D(w)\right]_\lambda + \delta[[r,p]_\lambda,w]_\lambda$
i.e. , D is a L. T $(\alpha,\beta)_1$-Hom Γ-derivation of $(A,[.,.]_\lambda,\theta_1,\theta_2)$

Corollary 2.7:
D is a L. T α- Hom Γ-derivation of $(A, [.,.]_\lambda, \theta_1, \theta_2)$ on Bi-Hom Γ- L algebra if and only if D is a J. T α-Hom Γ-derivation of $(A, [.,.]_\lambda, \theta_1, \theta_2)$.
Proof.
If D is a J. T α-Hom Γ-derivation of $(A, [.,.]_\lambda, \theta_1, \theta_2)$, then 1 follows immediately.
2 holds because

$$B(r,p,w) + B(p,w,r) + B(w,r,p)$$
$$= \theta_2 o \theta_1 \left(\left[\left[D(r), \alpha\left(\theta_1{}^k(p)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(r)\right), D(p) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(r)\right), \alpha\left(\theta_2{}^k(p)\right) \right]_\lambda, D(w) \right]_\lambda \right) + \delta[[r,p]_\lambda, w]_\lambda$$
$$+\theta_2 o \theta_1 \left(\left[\left[D(p), \alpha\left(\theta_1{}^k(w)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(r)\right) \right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(p)\right), D(w) \right]_\lambda, \alpha\left(\theta_2{}^k(r)\right) \right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(p)\right), \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda, D(r) \right]_\lambda \right) + \delta[[p,w]_\lambda, r]_\lambda$$
$$+\theta_2 o \theta_1 \left(\left[\left[D(w), \alpha\left(\theta_1{}^k(r)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(p)\right) \right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(w)\right), D(r) \right]_\lambda, \alpha\left(\theta_2{}^k(p)\right) \right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(w)\right), \alpha\left(\theta_2{}^k(r)\right) \right]_\lambda, D(p) \right]_\lambda \right) + \delta[[w,r]_\lambda, p]_\lambda$$
$$= \theta_2 o \theta_1 \left(\left[\left[D(r), \alpha\left(\theta_1{}^k(p)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(p)\right), \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda, D(r) \right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(w)\right), D(r) \right]_\lambda, \alpha\left(\theta_2{}^k(p)\right) \right]_\lambda \right)$$
$$+\theta_2 o \theta_1 \left(\left[\left[\alpha\left(\theta_1{}^k(r)\right), D(p) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[D(p), \alpha\left(\theta_1{}^k(w)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(r)\right) \right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(w)\right), \alpha\left(\theta_2{}^k(r)\right) \right]_\lambda, D(p) \right]_\lambda \right)$$
$$+\theta_2 o \theta_1 \left(\left[\left[\alpha\left(\theta_1{}^k(r)\right), \alpha\left(\theta_2{}^k(p)\right) \right]_\lambda, D(w) \right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(p)\right), D(w) \right]_\lambda, \alpha\left(\theta_2{}^k(r)\right) \right]_\lambda + \left[\left[D(w), \alpha\left(\theta_1{}^k(r)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(r)\right) \right]_\lambda \right) = 0,$$

Therefore D is a L.T α-HomΓ-derivation on Bi-Hom Γ- L algebra $(A, [.,.]_\lambda, \theta_1, \theta_2)$.

Theorem 2.8: D is a L. T $(\alpha,\beta)_2$- Hom Γ-derivation on Bi-Hom Γ- L algebra $(A, [.,.]_\lambda, \theta_1, \theta_2)$ if and only if D is a J. T $(\alpha,\beta)_2$-Hom Γ-derivation on Bi-Hom Γ- L algebra $(A, [.,.]_\lambda, \theta_1, \theta_2)$ such that

1- $$\left[\left[D\ (r), \alpha\left(\theta_1{}^k(p)\right) \right]_\lambda, (\beta - \alpha)\ \left(\theta_2{}^k(w)\right) \right]_\lambda = \left[\left[\alpha\left(\theta_1{}^k(r)\right), D\ (p) \right]_\lambda, (\beta - \alpha)\ \left(\theta_2{}^k(w)\right) \right]_\lambda$$

2- $\acute{B}(r,p,w) + \acute{B}(p,w,r) + B'(w,r,p) = 0$, Where $r,p,w \in A$
and $$\acute{B}(r,p,w) = \theta_2 o \theta_1 \left(\left[\left[D(r), \alpha\left(\theta_1{}^k(p)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(r)\right), D(p) \right]_\lambda, \beta\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[\beta\left(\theta_1{}^k(r)\right), \beta\left(\theta_2{}^k(p)\right) \right]_\lambda, D(w) \right]_\lambda \right)$$

Proof.
Let D be a L. T $(\alpha,\beta)_2$-Hom Γ-derivation on Bi-Hom Γ- L algebra $(A, [.,.]_\lambda, \theta_1, \theta_2)$ use the fact that

$D([[r,p]_\lambda,w]_\lambda) = -D([[p,r]_\lambda,w]_\lambda)$. As well as the fact that

$-D([[p,r]_\lambda,w]_\lambda) = -\left[\left[D(p),\alpha(\theta_1{}^k(r))\right]_\lambda,\alpha(\theta_2{}^k(w))\right]_\lambda$

$-\left[\left[\alpha\left(\theta_1{}^k(p)\right),D(r)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda$

$-\left[\left[\beta\left(\theta_1{}^k(p)\right),\beta\left(\theta_2{}^k(r)\right)\right]_\lambda,D(w)\right]_\lambda - \delta[[p,r]_\lambda,w]_\lambda$

$= \left[\left[D(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,\alpha\left(\theta_2{}^k(w)\right)\right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(r)\right),D(p)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda$

$+\left[\left[\beta\left(\theta_1{}^k(r)\right),\beta\left(\theta_2{}^k(p)\right)\right]_\lambda,D(w)\right]_\lambda + \delta[[r,p]_\lambda,w]_\lambda$

Then we have

$\left[\left[D(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,\alpha\left(\theta_2{}^k(w)\right)\right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(r)\right),D(p)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda$

$= \left[\left[D(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(r),D(p)\right)\right]_\lambda,\alpha\left(\theta_2{}^k(w)\right)\right]_\lambda,$

That is

$\left[\left[D(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,(\beta-\alpha)\left(\theta_2{}^k(w)\right)\right]_\lambda = \left[\left[\alpha\left(\theta_1{}^k(r),D(p)\right)\right]_\lambda,(\beta-\alpha)\left(\theta_2{}^k(w)\right)\right]_\lambda$

Since D is a L. T $(\alpha,\beta)_1$- Hom Γ-derivation of $(A,[.,.,.]_\lambda,\theta_1,\theta_2)$, we have

$\theta_2 o\theta_1\big(D([[r,p]_\lambda,w]_\lambda)\big) = B(r,p,w)$, Hence

$B(r,p,w) + B(p,w,r) + B(w,r,p)$

$= \theta_2 o\theta_1\big((D[[r,p]_\lambda,w]_\lambda) + D([[p,w]_\lambda,r]_\lambda) + D([[w,r]_\lambda,p]_\lambda)\big) =$

$D([[\theta_2(r),\theta_2(p)]_\lambda,\theta_1(w)]_\lambda + [[\theta_2(p),\theta_2(w)]_\lambda,\theta_1(r)]_\lambda + [[\theta_2(w),\theta_2(r)]_\lambda,\theta_1(p)]_\lambda) = 0$

Suppose that, conversely, that D is a (J. T. $(\alpha,\beta)_2$- Hom Γ-derivation on Bi-Hom Γ- L algebra $(A,[.,.]_\lambda,\theta_1,\theta_2)$ satisfying 1 and 2. Note that

$B'(p,r,w)$

$= \theta_2 o\theta_1\left(\left[\left[D(p),\alpha(\theta_1{}^k(r))\right]_\lambda,\alpha(\theta_2{}^k(w))\right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(p)\right),D(r)\right]_\lambda,\beta(\theta_2{}^k(w))\right]_\lambda +\right.$

$\left.\left[\left[\beta\left(\theta_1{}^k(p)\right),\beta\left(\theta_2{}^k(r)\right)\right]_\lambda,D(w)\right]_\lambda\right) + \delta[[p,r]_\lambda,w]_\lambda$

$= -\theta_2 o\theta_1\left(\left[\left[D(r),\alpha(\theta_1{}^k(p))\right]_\lambda,\beta(\theta_2{}^k(w))\right]_\lambda - \left[\left[\alpha\left(\theta_1{}^k(r),D(p)\right)\right]_\lambda,\alpha\left(\theta_2{}^k(w)\right)\right]_\lambda -\right.$

$\left.\left[\left[\beta\left(\theta_1{}^k(r)\right),\beta\left(\theta_2{}^k(p)\right)\right]_\lambda,D(w)\right]_\lambda\right) - \delta[[r,p]_\lambda,w]_\lambda$

$= -\theta_2 o\theta_1\left(\left[\left[D(r),\alpha\left(\theta_1{}^k(p)\right)\right]_\lambda,\alpha\left(\theta_2{}^k(w)\right)\right]_\lambda - \left[\left[\alpha\left(\theta_1{}^k(r)\right),D(p)\right]_\lambda,\beta\left(\theta_2{}^k(w)\right)\right]_\lambda -\right.$

$\left.\left[\left[\beta\left(\theta_1{}^k(r)\right),\beta\left(\theta_2{}^k(p)\right)\right]_\lambda,D(w)\right]_\lambda\right) - \delta[[r,p]_\lambda,w]_\lambda$

$= -\acute{B}(r,p,w)$

Similarly, we can obtain equations (1) and (2). The proof of the remainder is the same as A similar argument proves the following result the corresponding proof of Theorem2.5.

Theorem 2.9:

D is a L. T $(\alpha,\beta)_3$-Hom Γ-derivation on Bi-Hom Γ- L algebra $(A,[.,.]_\lambda,\theta_1,\theta_2)$ if D is a J. T$(\alpha,\beta)_3$-Hom Γ-derivation on Bi-Hom Γ- L algebra $(A,[.,.]_\lambda,\theta_1,\theta_2)$ such that.

1- $\left[\left[D(r),(\alpha-\beta)\left(\theta_1{}^k(p)\right)\right]_\lambda\alpha\left(\theta_2{}^k(w)\right)\right]_\lambda$

$= \left[\left[(\alpha-\beta)\left(\theta_1{}^k(r)\right),D(p)\right]_\lambda,\alpha\left(\theta_2{}^k(w)\right)\right]_\lambda$

2- $\acute{\acute{B}}(r,p,w) + \acute{\acute{B}}(p,w,r) + \acute{\acute{B}}(w,r,p) = 0$. Where $r,p,w \in A$ and

$$\dot{\dot{B}}(r,p,w) = \theta_1 o \theta_2 \left(\left[\left[D(r), \alpha\left(\theta_1{}^k(p)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[\beta\left(\theta_1{}^k(r)\right), D(p) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[\beta\left(\theta_1{}^k(r)\right), \beta\left(\theta_2{}^k(p)\right) \right]_\lambda, D(w) \right]_\lambda \right)$$

Proof.

Assume that D be a L. T $(\alpha,\beta)_3$- Hom Γ-derivation on Bi-Hom Γ- L algebra $(A,[.,.,.]_\lambda,\theta_1,\theta_2)$. Use the fact that. $D([[r,p]_\lambda,w]_\lambda) = -D([[p,r]_\lambda,w]_\lambda)$,

As well as the fact that

$$-D([[p,r]_\lambda,w]_\lambda)$$
$$= -\left[\left[D(p), \alpha\left(\theta_1{}^k(r)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda - \left[\left[\beta\left(\theta_1{}^k(p)\right), D(r) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda - \left[\left[\beta\left(\theta_1{}^k(p)\right), \beta\left(\theta_2{}^k(r)\right) \right]_\lambda, D(w) \right]_\lambda - \delta[[p,r]_\lambda,w]_\lambda$$
$$= \left[\left[D(r), \beta\left(\theta_1{}^k(p)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(r)\right), D(p) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[\beta\left(\theta_1{}^k(r)\right), \beta\left(\theta_2{}^k(p)\right) \right]_\lambda, D(w) \right]_\lambda + \delta[[r,p]_\lambda,w]_\lambda,$$ Then we have

$$\left[\left[D(r), \alpha\left(\theta_1{}^k(p)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[\beta\left(\theta_1{}^k(r), D(p)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda$$
$$= \left[\left[D(r), \beta\left(\theta_1{}^k(p)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[\alpha\left(\theta_1{}^k(r)\right), D(p) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda,$$ That is

$$\left[\left[D(r), (\alpha-\beta)\ \left(\theta_1{}^k(p)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda =$$
$$\left[\left[(\alpha-\beta)\ \left(\theta_1{}^k(r)\right), D(p) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda,$$ Since D is a (L. T) $(\alpha,\beta)_1$-Hom Γ-derivation of $(A,[.,.,.]_\lambda,\theta_1,\theta_2)$, we have

$\theta_2 o \theta_1\left(D([[r,p]_\lambda,w]_\lambda)\right) = B(r,p,w)$, Hence

$$B(r,p,w) + B(p,w,r) + B(w,r,p)$$
$$= \theta_2 o \theta_1\left((D[[r,p]_\lambda,w]_\lambda) + D([[p,w]_\lambda,r]_\lambda) + D([[w,r]_\lambda,p]_\lambda)\right)$$
$$= D([[\theta_2(r),\theta_2(p)]_\lambda,\theta_1(w)]_\lambda + [[\theta_2(p),\theta_2(w)]_\lambda,\theta_1(r)]_\lambda + [[\theta_2(w),\theta_2(r)]_\lambda,\theta_1(p)]_\lambda) = 0$$

conversely, Suppose that D is a (J. T. $(\alpha,\beta)_3$-Hom Γ-derivation on Bi-Hom Γ- L algebra $(A,[.,.]_\lambda,\theta_1,\theta_2)$ satisfying 1 and 2. Note that

$$\dot{\dot{B}}(p,r,w)$$
$$= \theta_1 o \theta_2 \left(\left[\left[D(p), \alpha\left(\theta_1{}^k(r)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[\left(\theta_1{}^k(p), D(r)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda + \left[\left[\beta\left(\theta_1{}^k(p)\right), \beta\left(\theta_2{}^k(r)\right) \right]_\lambda, D(w) \right]_\lambda \right) + \delta[[p,r]_\lambda,w]_\lambda$$
$$= -\theta_1 o \theta_2 \left(\left[\left[D(r), \beta\left(\theta_1{}^k(p)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda - \left[\left[\alpha\left(\theta_1{}^k(r), D(p)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda - \left[\left[\beta\left(\theta_1{}^k(r)\right), \beta\left(\theta_2{}^k(p)\right) \right]_\lambda, D(w) \right]_\lambda \right) - \delta[[r,p]_\lambda,w]_\lambda$$
$$= -\theta_1 o \theta_2 \left(\left[\left[D(r), \alpha\left(\theta_1{}^k(p)\right) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda - \left[\left[\beta\left(\theta_1{}^k(r)\right), D(p) \right]_\lambda, \alpha\left(\theta_2{}^k(w)\right) \right]_\lambda - \left[\left[\beta\left(\theta_1{}^k(r)\right), \beta\left(\theta_2{}^k(p)\right) \right]_\lambda, D(w) \right]_\lambda \right) - \delta[[r,p]_\lambda,w]_\lambda$$
$$= -\dot{\dot{B}}(r,p,w)\ .$$

Similarly, we can obtain equations (1) and (2). The proof of the remainder is the same as A similar argument proves the following result the corresponding proof of Theorem 2.5.

Remark 2.10:

Corollary 2.6 can also be concluded from the 2.7 or theorem 2.8 since for any , p , w ∈ A, $B(r,p,w) = \acute{B}(r,p,w) = \acute{\acute{B}}(r,p,w)$
When D is a J. T α- Hom Γ-derivation

Conclusions

In this research we have reached to D is a Lie triple $(\alpha,\beta)_i$ – Hom Γ-derivation if and only if D is a Jordan triple $(\alpha,\beta)_i$ Hom Γ-derivation on Bi-Hom Γ- Lie algebra $(A,[.,.]_\lambda,\theta_1,\theta_2)$ under some conditions, where $i = 1,2,3$.

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