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Lossless Encoding Method Based on a Mathematical Model and Mapping Pixel Technique for Healthcare Applications

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Abstract
The compression system typically comprises two fundamental categories: compression and decompression. Within the realm of compression, there exist two primary types: lossy compression and lossless compression. In this research, a novel lossless digital image encoding technique is presented. This method depends on performing a series of iterative mathematical transformations. The methodology of performing this method is similar to the process of performing the fractal compression algorithm, with an essential difference: the fractal pressure is classified as lossy compression, but this new method is lossless compression. It is well known that lossless compression methods produce higher-quality images than lossy methods, but they do not achieve as high a compression ratio as lossy methods. This research has overcome this problem. A high compression ratio has been obtained with images retrieved entirely according to the original (lossy). Lossless compression holds immense significance across a spectrum of applications, especially in critical domains such as healthcare, where the imperative is to retrieve images that are indistinguishable from the originals.

Keywords: fractal encoding, hourglass, lossy compression, lossless compression, mathematical model.

طريقة ترميز بدون خسارة تعتمد على نموذج رياضي وتقنية رسم خرائط للتطبيقات الصحية

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الخلاصة
يتكون نظام الضغط عادةً من فئتين أساسيتين: الضغط وإزالة الضغط. يوجد في عالم الضغط نوعان أساسيان: الضغط مع فقدان البيانات والضغط بدون فقدان البيانات. في هذا البحث ، تم تقديم تقنية جديدة لتشفير الصور الرقمية بدون فقدان البيانات .تعتمد هذه الطريقة على إجراء سلسلة من التحولات الرياضية المتكررة .منهجية تنفيذ هذه الطريقة مشابهة لمنهجية تنفيذ خوارزمية ضغط الكسوريات، مع وجود اختلاف جوهري ، يتم تصنيف ضغط الكسوريات على أنه ضغط ضياع ، ولكن هذه الطريقة الجديدة بلا خسارة . من

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المعروف جيدًا أن طرق الضغط بلا خسارة تنتج صورًا بجودة أعلى من طرق الضياع ، لكنها لا تحقق نسبة ضغط عالية مثل طرق الخسارة في البيانات .في هذا البحث تم التغلب على هذه المشكلة .تم الحصول على نسبة ضغط عالية مع الصور المسترجعة بالكامل وفقًا للأصل . يحتل الضغط بدون فقدان أهمية كبيرة عبر مجموعة واسعة من التطبيقات، خاصة في المجالات الحيوية مثل الرعاية الصحية، حيث يكون من الضروري استرداد الصور التي لا يمكن تمييزها عن النسخ الأصلية.

الكلمات المفتاحية : ترميز كسوري، ساعة رملية، ضغط مفقود، ضغط بلا فقدان، نموذج رياضي.

1. Introduction

The exponential growth in data handling, especially in the realm of digital images, has ushered in a transformative era across various disciplines. This applies to digital images as one of the types of data (text, audio, image, or video). There is a very wide circulation of images in many fields, such as disease diagnosis, space observation, meteorology, remote sensing, and computer vision [1].

In all these applications, the images need to be processed in order to extract useful information that will be used later to make the appropriate decision. Coding and compressing a digital image are two of the most important image processing operations, similar to other operations such as classification and segmentation. This field of science has received very wide attention from researchers, scientists, and software companies, with the aim of developing image processing applications [2].

Image encoding is a technique of data compression that encodes the entire image with limited bits by reducing the redundancy of the image and then storing or transmitting the data in an efficient form. The image, after decoding, can resemble the original picture as closely as possible. Figure 1 shows the general block diagram of the image compression system [3].

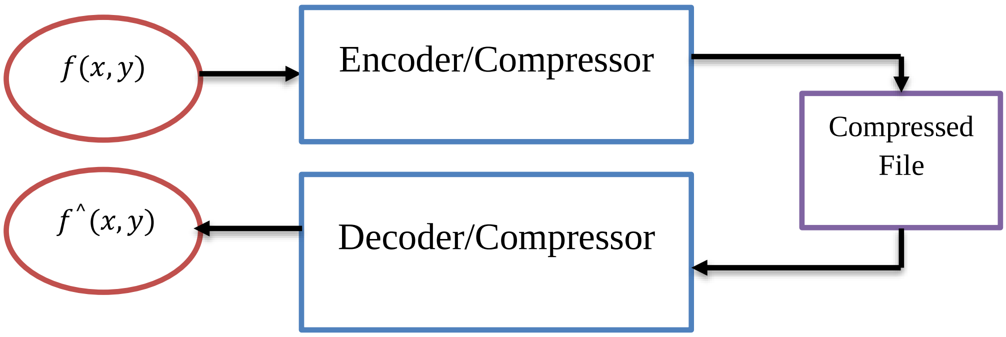

Figure 1: The general image compression system [1]

There are two categories of compression: Lossless compression, which is the process of compressing data so that when it is uncompressed, it will exactly match the original data, provides low data compression. This is appropriate for binary data, as it must be precisely recreated and decompressed, such as executables, documents, etc. The second category is lossy compression, where perfect recovery is not possible but provides large data compression. This type of compression is suitable for images, TV signals, teleconferencing, and music; it is not necessary to duplicate it exactly. For the majority of uses, an approximate representation of the original is sufficient [4] [5].

When compared to raw binary data encoding, encoding a picture is very different. This is so that encoders made specifically for images can take advantage of certain statistical characteristics that they possess. Additionally, a few of the image's finer elements could be sacrificed in order to free up a little bit more data [6].

When compared to raw binary data encoding, encoding a picture is very different. This is so that encoders made specifically for images can take advantage of certain statistical characteristics that they possess. Additionally, a few of the image's finer elements could be sacrificed in order to free up a little bit more data [7].

It is possible to summarize the objective of encoding (compression) as follows: [7]

- Uncompressed photos could take longer to load on websites.
- For sending and uploading images.
- Lowering the hard drive's essential storage impact.

Many lossless and lossy encoding (compression) methods have been implemented and developed over the years, all of which are directed towards achieving encoding speed and a high compression ratio with high-quality recovered images. But as it is known, obtaining high quality will be at the expense of compression ratio, and vice versa. The implementation of the idea presented in this research led to obtaining a very high compression ratio and a high-quality image [8].

2. Related Work

Numerous academics with extensive experience in the field of data and picture coding have given us a number of new research articles, methods, advances, and updates in this area, including:

- Paulo et al. [9] provided a technique to increase the usable scope of compression to lossy from lossless while enhancing the functionality of biomedical vision algorithms. To find mitochondria in pictures taken using an electron microscope, the authors employed a learning-based network. Authors offer two different sorts of new results. The first one allows for the use of up to 15 percent compression for a maximum detection loss of 3%. The second one allows for an increase in the compression range of up to 135 times with less than 5% missing.
- Wu et al. [10] used visually lossless encoding and proposed a method for converting healthy photos. In order to compress photos without causing any noticeable loss in image quality, a technique known as visually lossless coding must conceal obvious distortions. Based on the set split into hierarchical trees, the recommended structure was determined.
- Al-Khafaji et al. [11] presented a hierarchical autoregressive method for compressing images based on the idea of multi-layer modeling of associated coefficients of auto-regression, which is an adapted form of the model with a hierarchy of autoregressive. The test's outcomes indicate that the proposed technique improves compression ratio while preserving image superiority associated with predictive coding, traditional autoregressive models, and hierarchical autoregressive models on a sequence of certain images.
- Abed and Noaman [12] offered a new approach to developing encryption techniques based on the McLaurin series. A novel mathematical idea was used to encrypt data, increase its security while being transported over the internet, and store it secretly on a computer to avoid data eavesdropping. The McLaurin series is a one-way function.
- Al-din et al. [13] suggested a strategy dependent on howls, which are how wolves communicate with one another to find out where they are and to warn one another of impending hazards. In order to make it difficult for cryptanalytics to monitor the progress of

building the system, this paper proposes a new technique that divides the lettering of the message into groups and transfers keys between groups. It also uses a variety of cryptanalysis techniques to evaluate the proposed algorithm.

- U. Nandi [14] presented adaptive fractal techniques using linear affine maps during encoding that have limited pixel intensity approximation ability. In order to increase the image quality further, non-linear affine maps can be used that generalize the pixel intensity approximation and generate much better approximations.
- Yamagiwa et al. [15] proposed a method with principal component analysis (PCA) and a deep neural network (DNN) to predict the entropy of data to be compressed. The method infers an appropriate compression program in the system for each data block of the input data and achieves a good compression ratio without trying to compress the entire amount of data at once. This paper especially focuses on lossless compression for image data, focusing on the image blocks.

3. Image Compression Ratio (CR)

Image compression ratio is a measurement of the proportional size reduction of the image data representation brought about by the compression process. It is mathematically represented as the division of uncompressed size by compressed size [16]:

$$CR = uncompressed\ size\ /compressed\ size \qquad (1)$$

4. Error Metrics

The mean square error (MSE) and peak signal-to-noise ratio (PSNR) are two well-known error measures that are used to assess the different images in compression techniques. The peak error is measured by PSNR, whereas the cumulative squared error (MSE) is a comparison of the original and compressed pictures [16].

4.1 Mean Square Error (MSE)

The MSE measures the overall squared difference in error between the original and compressed image. Less error and good reconstruction quality are indicated by a lower MSE score. A definition of MSE is:

$$MSE = \frac{1}{mn}\sum_{y=1}^{m}\sum_{x=1}^{n}[I(x,y) - I'(x,y)]^2 \qquad (2)$$

Where: I(x,y) : original image,
I'(x,y) : recovered image,
M,N : image dimensions.

4.2 Signal to Noise Ratio (SNR)

The S/N is an additional way to state the SNR. The noise level is defined as the ratio of a signal's power to all other nearby electrical signals. In a statistical sense, SNR can also be defined as being equal to the mean divided by the standard deviation. A higher SNR value means a very good reconstructed image, and vice versa. So it must have as high an SNR value as possible. The following formula is used to compute the SNR of the retrieval image [17]:

$$SNR_{ms} = \frac{\sum_{y=1}^{m}\sum_{x=1}^{n} I'(x,y)^2}{\sum_{y=1}^{m}\sum_{x=1}^{n}[I'(x,y) - I(x,y)]^2} \qquad (3)$$

4.3 Peak Signal_To_Noise Ratio (PSNR)

It is the ratio of the maximum power of the original image compared to the power of the noise created as a result of compression. Because there are numerous signals with a wide range, PSNR is stated using the decibel scale (db). Better image quality is achieved with higher PSNR values. PSNR is defined using the mean squared error (MSE). The following formula is used to compute the PSNR of the retrieval image [18]:

$$PSNR = 20 * log10\,(255\,/\,sqrt(MSE)) \quad (4)$$

A greater PSNR number is desirable since it indicates a stronger signal-to-noise ratio. The original image acts as the signal, while any error in the retrieved image serves as noise. Consequently, a superior compression method with a lower MSE (and a high PSNR) [18].

5. Methodology

As it is known, lossyless compression algorithms provide a high-quality recovered image but a lower compression ratio than lossy methods, which provide a higher compression ratio but lower image quality. The method described in this study can be said to combine the benefits of lossy and lossless. According to the hourglass algorithm (a set of transformations), any data vector can be reduced to a single integer. The values of four variables must be kept constant during this operation. In the end, five integer numbers must be saved in order to retrieve the same row of data.

Once two-dimensional digital images have been combined into a single data vector, it is possible to use this process (the hourglass) to encode (compress) digital images. Figure 2 depicts the overall model for the suggested technique.

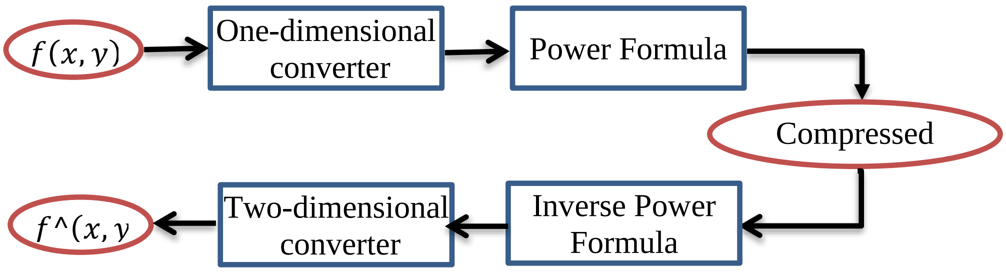

Figure 2: General Diagram of the Proposed Method

The row of data is then cut in half in each loop, following a series of nested recursive math operations, until the last loop yields a single number. This number is an illustration of the compressed image. Four more values will be required to gradually undo the transformations and recover the data row that is exactly the same as the original, so as to undo the compression process and retrieve the original image:

- no. of round (n)
- length of data (m)
- Required data (k,a)

5.1 Encoding Data using the Power Function Formula

The power function f(x) is represented by the below formula [19] [20]:

$$f(x)=k\,(xn) \qquad (5)$$

Where:

x: any variable,
k: a constant that is not equal to 0,
n: a real number.

Whereas there are many formulas for the power equation, it is possible to select any formula to encode or decode any data vector by adopting three basic phases (requirement data, encoding phase, and decoding phase). For example, C=2x.3y, where c is the output, x is the first element, and y is the second element in the vector [20].

5.2 Power Function Applications

Spectrum analysis, radio, audio, and light applications are among the significant uses of power series in engineering. For instance, it is possible to perform this analytically using a power series called the Fourier series (Fourier transform). This study adds an additional use for this kind of potent function in the field of image processing—image encoding. Figure 3 simplifies the concept of the proposed method and the use of the power formula, which in each iteration led to reducing any data vector to half its size. Figure 3 simplifies the concept of the proposed method [20].

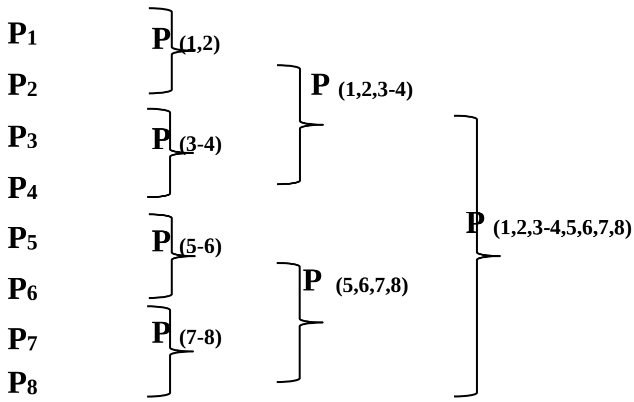

Figure 3: The concept of the proposed method

Several equations are used within the Power Function to encode row data: -

$$P = (x, y) \rightarrow 2^x . 3^y \tag{6}$$

$$P1 = P \ mod \ 26 \tag{7}$$

$$P2 = P1 . k^a \ mod \ k^a . 26 \tag{8}$$

It is possible to summarize the steps of encoding row data using the Power Function formula. Figure 4 shows the flowchart of the required steps.

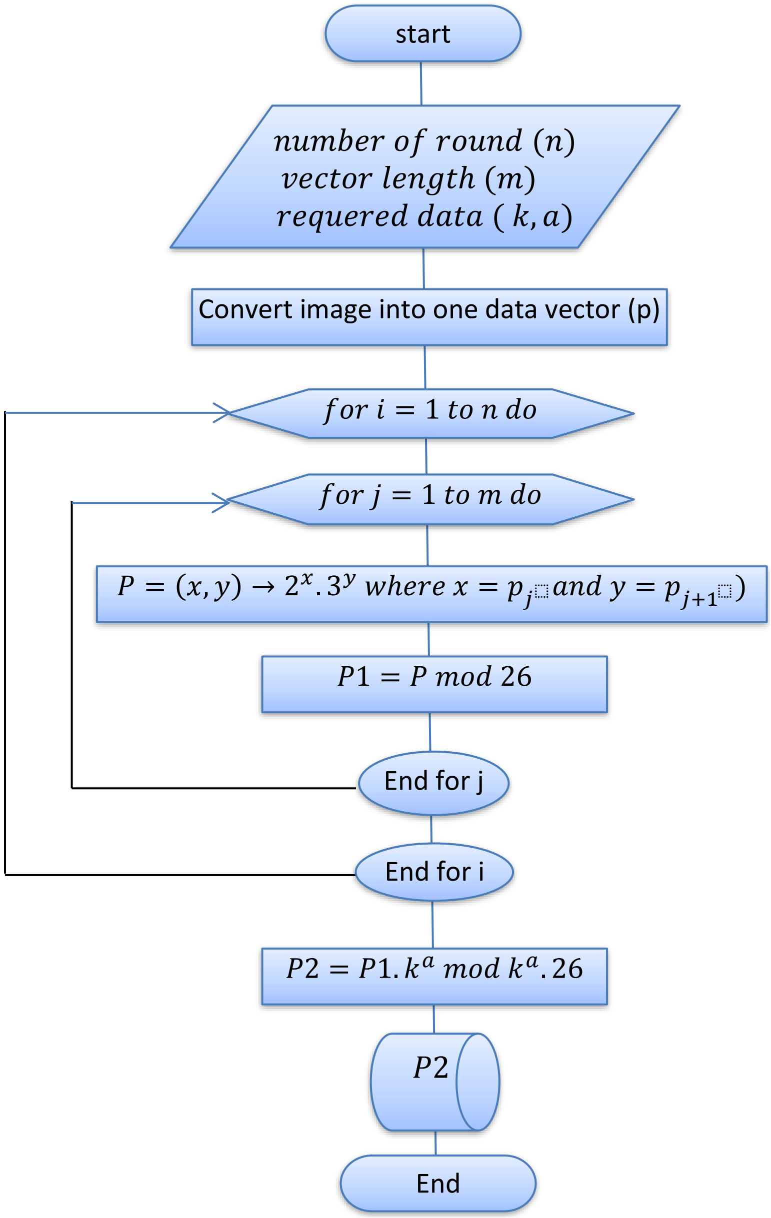

Figure 4: Flowchart of the Proposed Encoding Row Data Technique

5.3 Decoding Phase

The process of recovering the compressed image is carried out by performing the compression operations in reverse order, and this is the most important characteristic of lossless methods. The last process that finishes in the decompression phase will be the first process in the decompression phase. In order to reverse the encoding process, additional operations must be executed to get the original data row. The last process that is finished in the compression phase will be the first process in the decompression phase, as shown in the following flowchart. Figure 5 shows the steps of the proposed decoding row data technique.

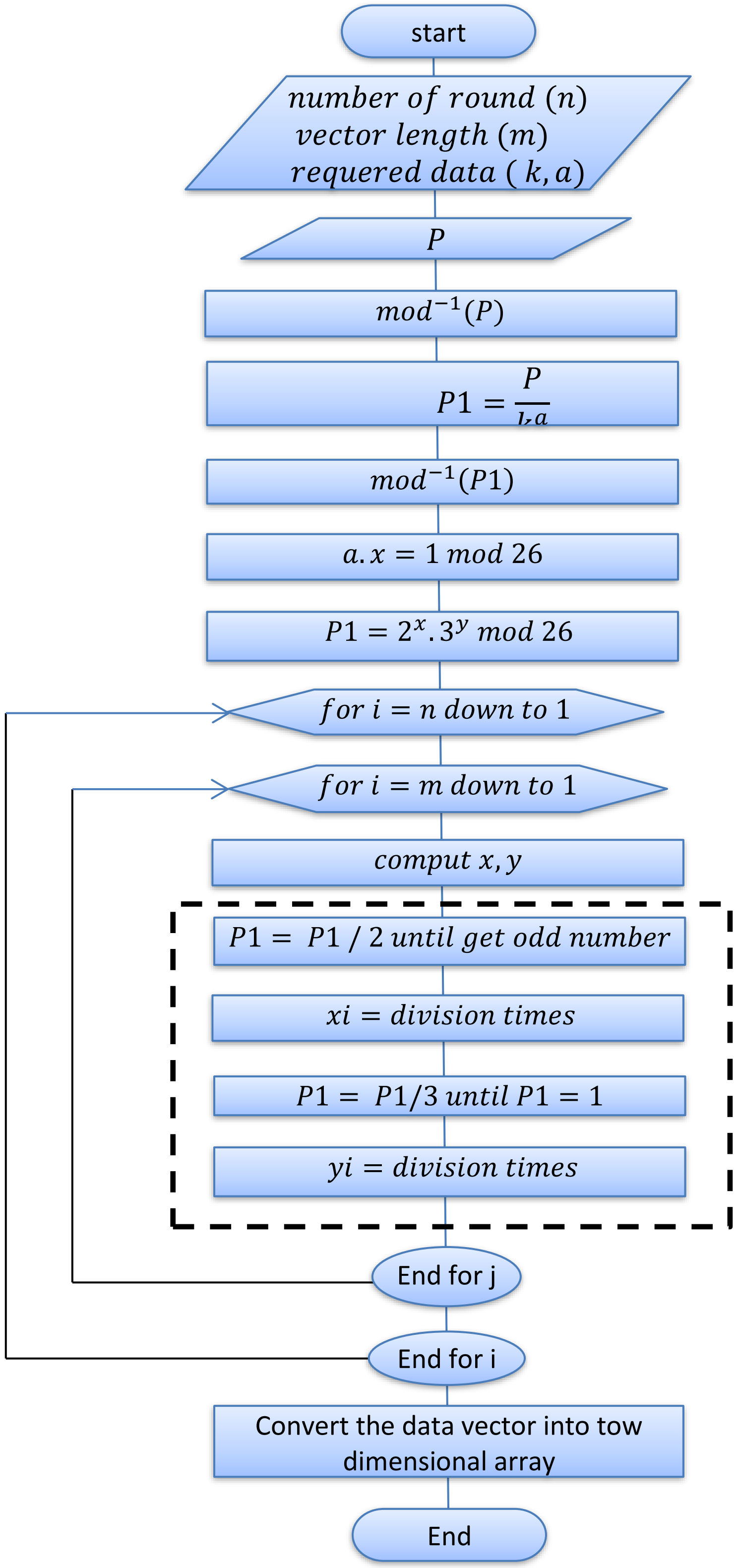

Figure 5: Flowchart of the Proposed Decoding Row Data Technique

This is comparable to compression using the fractals method, which divides the image into various regions (based on specific criteria), each of which will represent a portion of the image. A digital picture lossy encoding technique based on fractals is called fractal encoding. The technique, which relies on the fact that sections of a picture frequently resemble other parts of the same image, is particularly well suited for textures and nature photographs. These components are transformed by fractal algorithms into mathematical information known as fractal codes, which are then utilized to reproduce the encoded image. [21]. According to the fractal compression algorithm, using affine transformations, the data from each region will be reduced to just 5 values (each part of the image (10 x 10) will be replaced by 5 digits). This will be utilized subsequently to acquire the information for that image segment. [22] [23]: -

The coordinates (x,y)
The scale (s)
The offset value (o)
The symmetry state (sy)

6. Result and Discussion

This study has shown that the power function may be used in the context of digital image processing and picture compression. A decoding and encoding algorithm are created. It is accurate to suggest that the power function may be used in this field as well as in other ones like spectrum analysis, radio, audio, and lighting.

To illustrate, consider a digital image sized at 50 x 50, which has been partitioned into 25 individual regions, each measuring 10 x 10 pixels, as depicted in Figure 6.

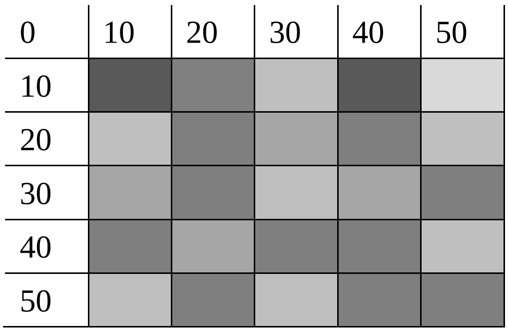

Figure 6: The digital image, originally 50 x 50 pixels, has been divided into 25 regions, each 10 x 10 pixels in size.

In the context of the fractal compression algorithm utilizing affine transformations, the 10 by 10 pixel sections within the image are substituted with five numerical values. Prior to compression, the original image occupied a space equivalent to 50 x 50 pixels, resulting in a size of 2.44 kilobytes (kb). Fractal compression mandates the use of five numeric values to retrieve each component of the image accurately. Consequently, with 25 zones, each measuring 10 by 10 pixels, the aggregate data required for decompression and image recovery stands at a mere 0.122 kb. This calculation leads to a compression ratio of 2.44 to 0.122, yielding a value of 20. In line with the proposed approach based on a power function formula, the entirety of the image is condensed into a mere five integer values. These five integers suffice for restoring the original image's dimensions (50 x 50). Consequently, the compression ratio can be ascertained as 2500/5, resulting in an impressive compression ratio of 500.

In the previous example, by applying the proposed method to each part of the image, as in fractal compression of a fixed-part image, the size of the compressed image can be calculated by multiplying the number of partitions by 5. In this case, the same compression ratio is obtained but with a substantial difference, which is without loss rather than loss. In order to clarify the results of the proposed method, it is possible to do this by making a comparison between it and fractal pressure, as it is the closest method to it in terms of the way it works. Some essential differences between the proposed approach and the fractal algorithm can be seen in Table 1.

Table 1: Key Distinctions between the Proposed Method and Fractal Compression

	Fractal encoding	**Proposed method**
1	Lossy compression type: The recovered data is not completely the original.	Lossless compression type: The recovered data is completely identical to the original.
2	It depends on dividing the image into several parts (partitions) and then using the affine transformation.	Convert the image to a data vector, and then use the power function.
3	The size of the compressed image depends on the number of partitions.	The number of divisions has no bearing on the size of the compressed picture.
4	As there are more divisions, the compression ratio decreases.	The original image size and the compression ratio are directly correlated.

Evidently, the application of the proposed method results in a consistent compressed image size of 5 bytes, irrespective of the original image's dimensions. Table 2 succinctly encapsulates this distinction. Furthermore, it's worth noting that the time required for both compression and decompression is directly correlated with the size of the image.

Table 2: Differences between the proposed method and fractal compression

Criterion	**Fractal Compression**	**The proposed method**
C.R.	Low compression ratio	high compression ratio
MSE	> 0	0
File size	5 byte x number of image partitions	5 byte

7. Conclusion

This research has introduced an innovative mathematical model that effectively encodes digital images using iterative transformations based on the power equation. This technique relies on condensing image data into a set of compact numerical representations, enabling the subsequent retrieval of the original image data with perfect fidelity, as indicated by an MSE of 0. MSE In the proposed method, it is always zero. In the fractal method, it is always greater than zero due to the presence of loss. The results have demonstrated remarkable compression ratios without compromising the accuracy and reliability of the data, which is especially vital when dealing with sensitive medical documents. The potential of this approach extends beyond image compression, as it could potentially be adapted to audio and video data as well. By expanding the method to encompass other media formats, such as audio and video, it has the power to revolutionize data compression techniques across various fields and industries. Further exploration and experimentation in this area hold significant promise for advancing the field of multimedia data processing and storage to new heights.

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