DOI: 10.24996/ijs.2020.61.2.25

ISSN: 0067-2904

Analytical Study of Earth Tides on Low Orbits Satellites

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Received: 31/3/ 2019 Accepted: 18/ 8/2019

Abstract

The main objective of this paper is to calculate the perturbations of tide effect on LEO's satellites (GEO11, GEO12, GEO13, NOAA2 and NOAA3). In order to achieve this goal, the changes in the orbital elements which include the semi major axis (a) eccentricity (e) inclination (i), right ascension of ascending nodes (Ω), and fifth element argument of perigee (ω) must be employed. In the absence of perturbations, these element remain constant. The results show that the effect of tidal perturbation on the orbital elements depends on the inclination of the satellite orbit. The variation in the ratio ($\Delta i/i$) decreases with increasing the inclination of satellite, while it increases with increasing the time.

Keywords: Tide perturbation, Orbital elements, Deformation of earth, LEO's.

دراسة تحليلية لتيارات الأرض على الأقمار الصناعية ذات المدارات المنخفضة

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الخلاصة

الهدف الرئيسي لهذا البحث هو دراسة تأثير اضطرابات المد والجزر على الأقمار الاصطناعية الواطئة (GEO11, GEO12, GEO13, NOAA2 , NOAA3) ولدراسة اضطرابات المد والجزر على هذه الأقمار تم أجراء الحسابات على العناصر المدارية للأقمار الصناعية والمتضمنة نصف المحور الكبير للمدار (a) الشذوذ المركزي (e) ميل المدار (i) الإزاحة الزاوية لنقطة الحضيض (Ω) مطلع المستقيم للعقدة الصاعدة(ω) هذه العناصر تبقى ثابتة بغياب الاضطرابات. يتبين من النتائج أن اضطرابات المد والجزر تعتمد على ميل المدار (i) وأن نسبة التغير ($\Delta i/i$) تقل بزيادة ميل المدارفي حين تزداد بزيادة فترة دورانه حول الارض.

Introduction

Perturbations are deviations from a normal, idealized, or unperturbed motion. Tthe most accurate method to analyze perturbations is via numerical analysis. The actual motion will vary from the theoretical two-body path due to perturbations caused by other bodies (such as the Sun and Moon) and additional forces not considered in Keplerian motion (such as a Non-spherical central body and drag). There are three main approaches to examine the effects of perturbations, namely the special perturbation techniques (using numerical methods), general perturbation techniques (using analytical methods), and semi analytical techniques. Perturbations forces include the accelerations resulting from the central body, drag, third body, solar-radiation pressure and many other forces affecting the satellite's orbit, but most are very small and are usually neglected. Tide effects are becoming more important as the expanding computational resources allowed to consider them as the perturbations

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force [1]. Tidal forces cause obvious changes in the low earth satellite orbit. The tidal effect is caused by the variations in the external gravitational force by celestial bodies; the force is known as the tidal force. At any point within or on the surface of earth, the external gravitational force by the celestial body can be split into two components, the first is equal to the gravitational force effect at the center of mass and the second is equal to the remainder. The tidal force tries to deform the equipotential surface so that the shapes will be elongated in the direction of the resultant force exerted by the celestial bodies (moon and sun). Astronomical observation imply that the solar potential is about 46% of the lunar potential. Table-1 shows the relative tidal potential from different celestial bodies.

Table 1-Relative contributions to tidal potential from different celestial bodies [2]

body	Tidal Potential
Moon	1.0
Sun	0.4618
Venus	0.000054
Jupiter	0.0000059
Mars	0.0000010

The tidal potential can be classified into three types, namely the tidal gravity variation g_1, the tidal tilt θ_t of the equipotential surface which is related to tidal gravity, and the tidal uplift of equipotential surface (Figure-1 [2]).

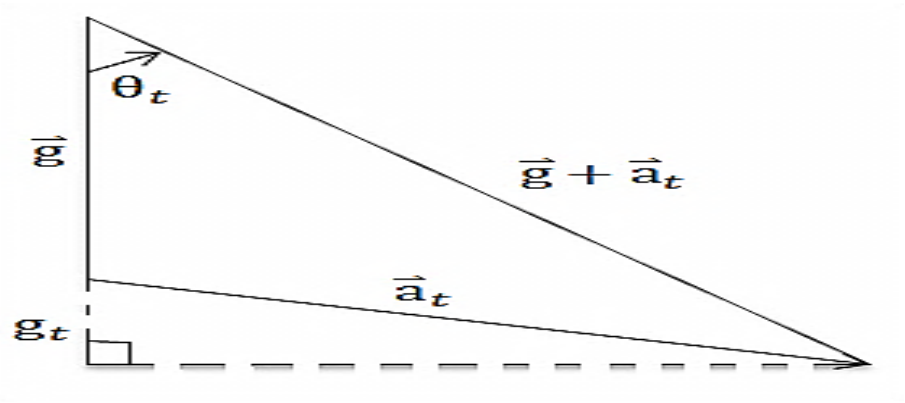


Figure 1-Tidal tilt [2].

In astrodynamics, two main types of tides were analyzed which are the solid-Earth and the oceanic tides. Although the two forms are different, they stem from many of the same sources. Solid-Earth tides are the deformations of the Earth due to perturbing forces. Internal forces result from the Earth's interior structure and involve complex models of the motions of the liquid and solid properties of the matter within the Earth. These forces are in the beginning to be explored [1]. Earth tides (also known as body tides, Ray1998) are more straightforward to calculate and do not require a model as the ocean tide. Instead, it is calculated from the astronomical tide-generating potential (i.e., the direct gravitational attraction of the Moon and Sun on Earth [3, 4]. The periodic tidal deformations of the Earth give rise to small but significant perturbations in the motions of close satellites, as pointed out by Kaula (1962). First attempts at analyzing orbits for these perturbations were reported by Newton, R. R. (1965, 1968) and Kozai (1968). The formers, in 1968, analyzed Doppler observations of four satellites collected by the Tranet network, while the latter analyzed camera observations of three satellites collected with the Smithsonian Baker-Nunn network**.** However**,** more studies by Anderle (1971), Smith, Kolenkiewicz & Dunn (1973) and Douglas, Klosko, Marsh & Williamson (1974) resulted in values for the Love numbers k_2 of 0.25 and even smaller. Lambeck & Cazenave (1973) pointed out that this apparently aberrant result was mainly the consequence neglecting the ocean tide in these earlier studies**.** Kaula (1969) dependence made a generalized development of Love numbers in terms of latitudinal and longitudinal dependencies. Lambeck, Cazenave & Balmino (1974) developed

such a theory and gave a general review of the solid and fluid tidal effects on close Earth satellite orbits [5]. Earth tides were calculated from the tide-generating potential by. Goring and Walters [4].
The effects of tidal deformation of earth, due to the sun and the moon, on close earth satellites were discussed by Yoshihide Kozai (1965**).** These effects are about ten percent of the direct luni-solar gravitational perturbations, and it is found that the eccentricity as well as the semi-major axis are not perturbed by the tides when the short-periodic terms are neglected. Lambeck (1974)reviewed the earth's tidal deformations which cause perturbations in the motions of close earth satellites, observations of which give estimates of the Love number k_2 and phase lag δ (In classical definition the Love number defines the response of a radially symmetric perfectly elastic earth to the perturbing potentials) . The earth's periodic tidal deformations were reviewed by Slichter (1972) for the solid earth tides [6]**.**

The main objective of this paper is to determine the tidal effects on Low Earth Orbits (LEOs) satellites, which are considered in satellite geodesy as mostly circular. Typically, they may accommodate gravity field missions and are also used for communication satellite constellations such as Global star and Iridium. The orbital period at these altitudes varies between 90 minutes and two hours. The radius of the satellite footprint (i.e. the area on the surface from where the satellite is visible above the horizon) is rather small and varies between 2000 and 4000 km. Geostationary Earth Orbits (GEOs) are mainly used for communication satellites because the orbits are very stable, while LEOs are required for global coverage [7].

In order to study the tidal effects on LEO satellites, the position and velocity are described while changes in the orbital elements must be employed. This includes determining the shape of the orbital semi major axis, a, eccentricity, e, inclination, i, and longitude of the pericenter or argument of perigee.

In the absence of perturbation, these elements remain constant. The remaining elements that determine the location of the satellite along its orbit can be described here by the mean anomaly, M, time, t, true anomaly, v, eccentricity, e and right ascension of ascending nodes, Ω.

Tidal Perturbation Theory

The expression "perturbed motion" implies that there is an unperturbed motion. In Celestial Mechanics the unperturbed motion is the orbital motion of two spherically symmetric bodies represented by the equations of motion, the solution of which is known in terms of simple analytical functions [8].
The basic Keplerian equation of motion is

$$\ddot{r} = \frac{GM}{r^3} r + k_s \qquad \cdots\cdots\cdots (1)$$

where r being the geocentric position vector of the artificial satellite.

G is the universal constant of gravitation $G = (6.67259 \pm 0.00085) \cdot 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$.

k_s *is the* perturbing vector and *M* is the mass.

The integration of this equation formally gives the solution

$r(t) = r(t\,;\, a_1, \dots, a_6), \qquad \dot{r}(t) = \dot{r}(t\,;\, a_1, \dots, a_6)$

with $a_1, \dots, a_6$ being free selectable integration constants. Preferably, the Kepler elements $a, e, i, \omega, ., M$ are used.

In reality, a certain number of additional forces act on the near-Earth satellite. To distinguish them from the central force (*central body acceleration*) these are called the *perturbing forces*. The satellite experiences additional accelerations because of these forces, which can be combined into a resulting *perturbing vector* k_s . The extended equation of motion is applied in equation (1).

k_s are perturbing forces that are in particular responsible for:

1. Accelerations due to the non-spherical and inhomogeneous mass distribution within Earth (central body), $\ddot{\boldsymbol{r}}_E$.
2. Accelerations due to other celestial bodies (Sun, Moon and planets), mainly $\ddot{\boldsymbol{r}}_S, \ddot{\boldsymbol{r}}_M$.
3. Accelerations due to Earth and oceanic tides, $\ddot{\boldsymbol{r}}_e, \ddot{\boldsymbol{r}}_O$.
4. Accelerations due to atmospheric drag, $\ddot{\boldsymbol{r}}_D$.
5. Accelerations due to direct and Earth-reflected solar radiation pressure, $\ddot{\boldsymbol{r}}_{SP}, \ddot{\boldsymbol{r}}_A$.

The perturbing forces causing the above effects in 1 to 3 are gravitational in nature, whereas the remaining forces are non-gravitational. The total force is

$k_s = \ddot{\boldsymbol{r}}_E + \ddot{\boldsymbol{r}}_S + \ddot{\boldsymbol{r}}_M + \ddot{\boldsymbol{r}}_e + \ddot{\boldsymbol{r}}_O + \ddot{\boldsymbol{r}}_D + \ddot{\boldsymbol{r}}_{SP} + \ddot{\boldsymbol{r}}_A \cdots\cdots\cdots (2)$

Figure 2 provides an illustrative description of the perturbing forces and accelerations. The resulting total acceleration depends on the location $\boldsymbol{r}$ of the satellite, i.e. a quantity which first has to be determined from the solution of the equation 1 as a function of time. The perturbed satellite motion can be interpreted to be a Keplerian motion with time variable elements of $a(t), e(t), i(t), \omega(t), (t), \vec{M}(t)$.

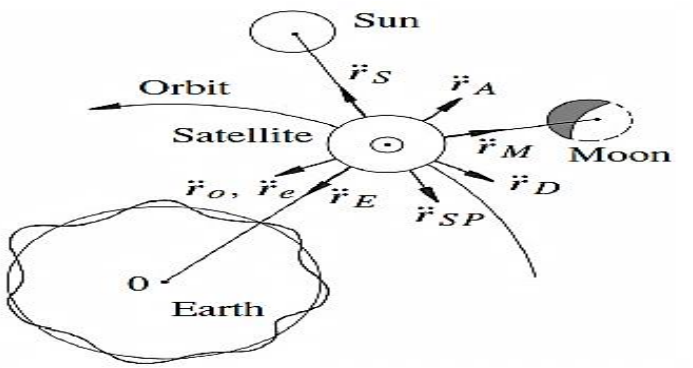


Figure 2-Perturbation forces acting on a satellite [7]

The appropriate basic equations that were formulated by Lagrange are the relation between the acting perturbing forces and the time dependent variations of the orbital elements. The total energy of the satellite motion is determined by

$$E_M = \frac{v^2}{2} - \frac{GM}{r} = -\frac{GM}{2a} \quad \cdots\cdots\cdots (3)$$

The negative term of the total energy, $GM/2a$, is also named the force function F.

With the potential V as the negative value of the potential energy and the symbol T for the kinetic energy, we find the following form of the force function

$$F = V - T \quad \cdots\cdots\cdots (4)$$

In a non-central force field

$$V = \frac{GM}{r} + R \quad \cdots\cdots\cdots (5)$$

$$F = \frac{GM}{r} + R - T = \frac{GM}{2a} + R \quad \cdots\cdots\cdots (6)$$

The function R contains all the components of V excluding the central term GM/r, and is called the perturbation function or disturbing potential.

For the purpose of completeness, an alternative form of the equation of motion in a non-central force field is given [7]

$$\ddot{r} = \text{grad}\, V = \nabla V \quad \cdots\cdots\cdots (7)$$

With Lagrange's perturbation equations, a relationship between the disturbing potential R and the variation of the orbital elements is established [9, 10].

$$\frac{da}{dt} = \frac{2}{na}\frac{\partial R}{\partial \vec{M}} \quad \cdots\cdots\cdots (8)$$

$$\frac{de}{dt} = \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial \vec{M}} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \omega} \quad \cdots\cdots\cdots (9)$$

$$\frac{d\omega}{dt} = -\frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i} + \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e} \quad \cdots\cdots\cdots (10)$$

$$\frac{di}{dt} = -\frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \omega} - \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \Omega} \quad \cdots\cdots\cdots (11)$$

$$\frac{d\Omega}{dt} = -\frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i} \quad \cdots\cdots\cdots (12)$$

$$\frac{d\vec{M}}{dt} = n - \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial e} - \frac{2}{na}\frac{\partial R}{\partial \text{a}} \quad \cdots\cdots\cdots (13).$$

Where R is the perturbing potential cf, with the origin of the coordinate system is transferred to the center of mass of the primary

$$R = \frac{GM}{r}\left(\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n (\mathrm{C}_{nm}\cos m\lambda + S_{nm}\sin m\lambda)P_{nm}(\cos\vartheta)\right) \quad \cdots\cdots\cdots (14)$$

The terms with $n = 1$ and $m = 0, 1$ become zero. The coefficients are named zonal when $m = 0$, tesseral when $n = 0$, and sectorial when $m = n$.

In the *first step* the disturbing potential

$$R = \sum_{n=2}^{\infty}\sum_{m=0}^{n} R_{nm} \quad \cdots\cdots\cdots (15)$$

is re-formulated as a function of the orbital elements:

$$R_{nm} = \frac{GMa_e^n}{a^{n+1}}\sum_{p=0}^{n} F_{nmp}(i)\sum_{q=-\infty}^{+\infty} G_{npq}(e)S_{nmpq}\left(\omega, \vec{M}, \Omega, \Theta\right) \quad \cdots\cdots\cdots (16)$$

Θ is the Greenwich sidereal time.

Result and Discussion

The equations of motion used in astrodynamics are not simple to solve because they're usually coupled systems of first-order or second-order, nonlinear, differential equations which have resisted direct solutions over the last 300 years. However, modern computers allow us to use numerical techniques.

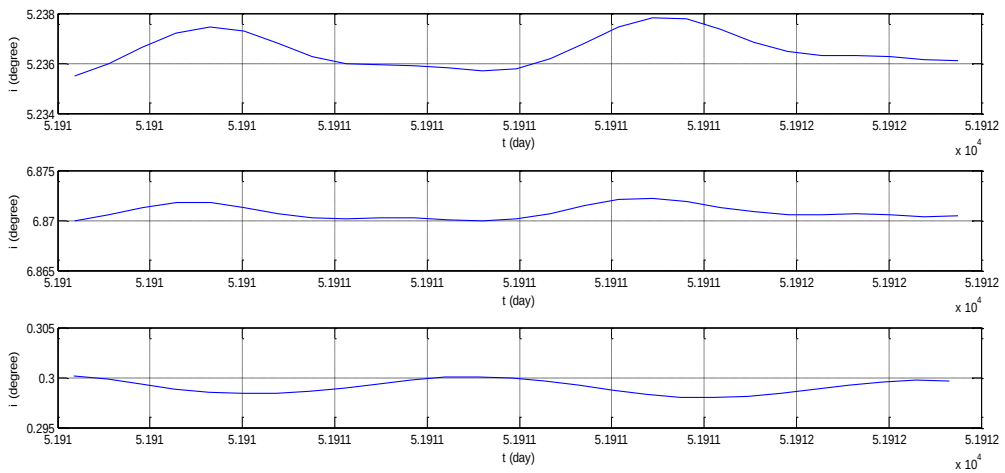
Numerical methods are distinguished by their simplicity and universal applicability when compared with analytical methods. With the use of the modern computer techniques the numerical effort only plays a minor role. This is why numerical methods are now used almost exclusively for orbit computations in satellite geodesy.

The osculating orbital elements for GEOs satellite are given in Table-2. Figures- 3-5 show the variation in the inclination of the above satellites due to the perturbation of the central body, including tidal of the Solid Earth over a short period (2 days) corresponding to about 25 revolutions around the earth with long periods of 100, 200 and 360 days. Note that the time axis in in the figures is given by the Julian date.

Table 2- Osculating orbital elements for GEO satellites.

GEO (11)	GEO (12)	GEO (13)
$i = 5.2366$ $\Omega = 72.6334$ $e = 0.0007115$ $w = 167.8990$ $a = 7389$	$i = 6.8667$ $\Omega = 58.9713$ $e = 0.0010975$ $w = 226.3761$ $a = 7401$	$i = 0.3022$ $\Omega = 104.9249$ $e = 0.0000989$ $w = 6.2494$ $a = 7345$

Figure 3-Change in inclination with time of GEO 11, GEO 12 and GEO 13 for 2 days**.**



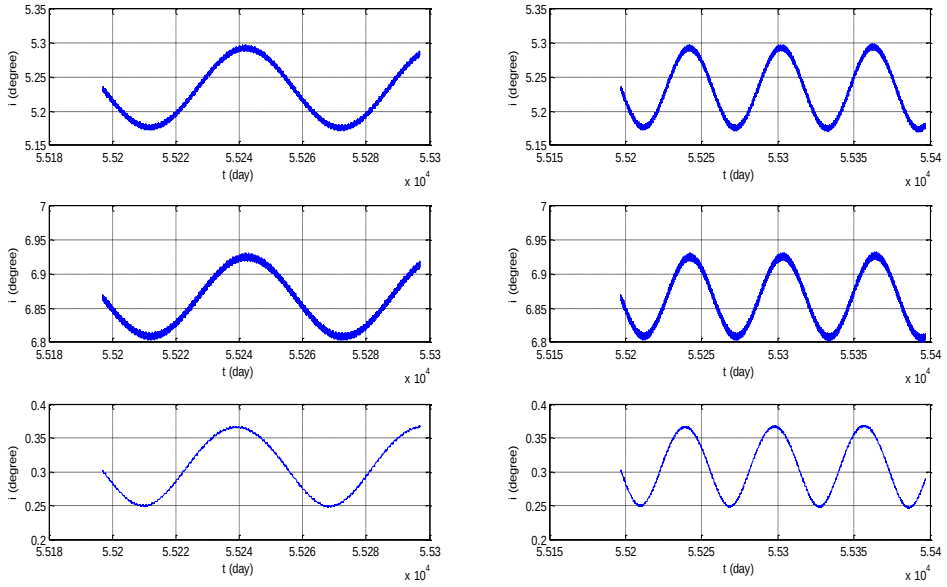


Figure 4-Change in inclination with time of GEO 11, GEO 12 and GEO 13.

Left panel: Change for 100 day, right panel: changes for 200 day.

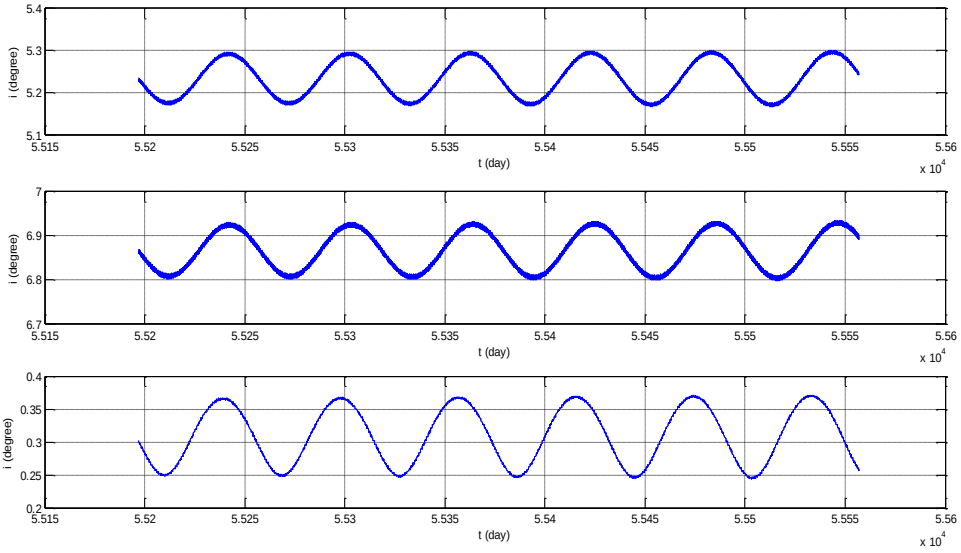


Figure 5-Change in inclination with time of GEO 11, GEO 12 and GEO 13 for 1year.

From these Figures -(3-5) we calculate the variation $(\Delta i/i)$. The results given in Table- 3 show that as the inclinations of satellites increase the $(\Delta i/i)$ values decrease.

Table 3-Change in inclination of GEOs satellites for different times

Satellite	$i(degree)$	**2 day** $(\Delta i/i)$	**100 days** $(\Delta i/i)$	**200 days** $(\Delta i/i)$	**360 days** $(\Delta i/i)$
GEO 11	5.2366	0.000577	0.02291	0.02291	0.02291
GEO 12	6.8667	0.000524	0.0262	0.0262	0.01747
GEO 13	0.3022	0.00993	0.36399	0.36399	0.36399

Figures- 6-7 show the periodic perturbation on the semi major axis and that the variation in $(\Delta a/a)$ remains constant over periods of 100, 200 and 360 days, as given in Table- 4.

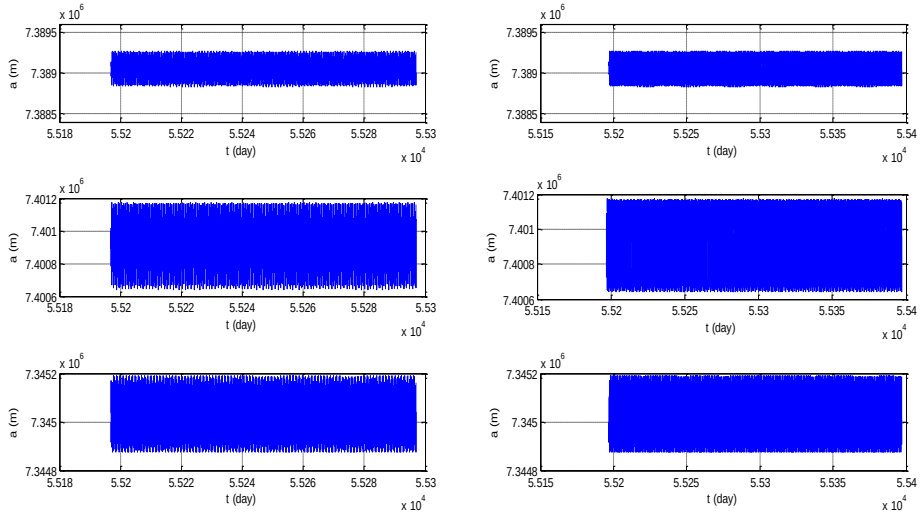


Figure 6-Perturbations on the semi-major axis of GEO 11, GEO 12 and GEO 13 Left panel: changes for 100 days, right panel: changes for 200 days.

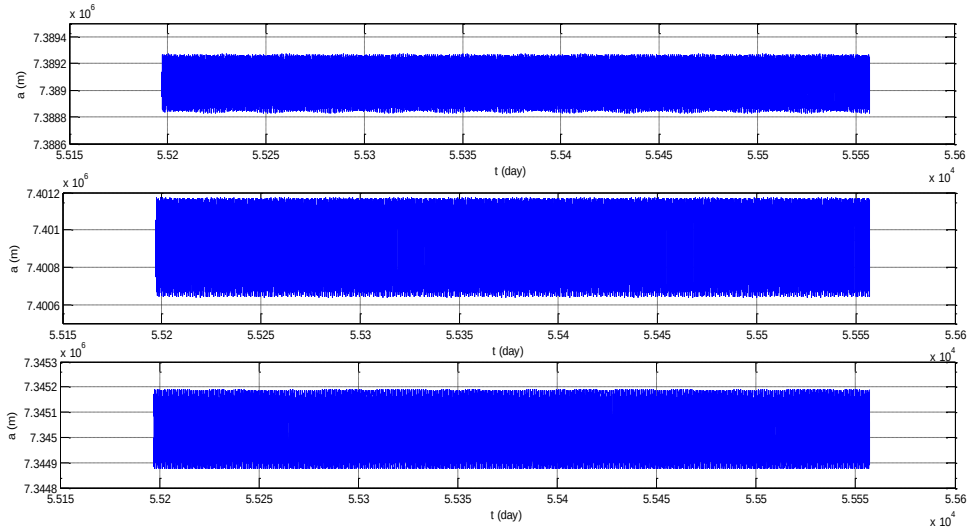


Figure 7-Perturbations on the semi-major axis of GEO 11, GEO 12 and GEO 13 for 1 year.

Table 4-Changes in the semi-major axis of GEO satellites for different times

Satellite	$a(Km)$	**2 day** $(\Delta a/a)$ ***10^{-5}**	**100 days** $(\Delta a/a)$ ***10^{-5}**	**200 days** $(\Delta a/a)$ ***10^{-5}**	**360 days** $(\Delta a/a)$ ***10^{-5}**
GEO 11	7389	0.257	5.413	5.413	5.819
GEO 12	7401	0.2297	7.161	7.161	7.161
GEO 13	7345	0.2723	3.812	3.812	4.084

Figure-8 shows the perturbation effect on other satellites (NOAA2 and NOAA3) over short periods (2, 6 and 10 days), while their orbital elements are given in Table- 5. We found that the inclination of the satellite orbit plays an important role in the effect of perturbation. The variation in $(\Delta i/i)$ given in Table- 6 shows a decrease as the inclination of the satellites increases.

Table 5-Osculating orbital elements for NOAA's satellites

NOAA (2)	**NOAA (3)**
$i = 101.7882$	$i = 101.9573$
$\Omega = 359.2863$	$\Omega = 263.1995$
$e = 0.0031771$	$e = 0.0006182$
$w = 14.3097$	$w = 196.7594$
$a = 6450$	$a = 6500$

Table 6-Change in inclination of NOAA's satellites for different times

Satellite	$i(degree)$	2 day $(\Delta i/i)$	6 day $(\Delta i/i)$	10 day $(\Delta i/i)$
NOAA 2	101.7882	0.00006877	0.0000786	0.00008841
NOAA 3	101.9573	0.00005688	0.0000687	0.00005884

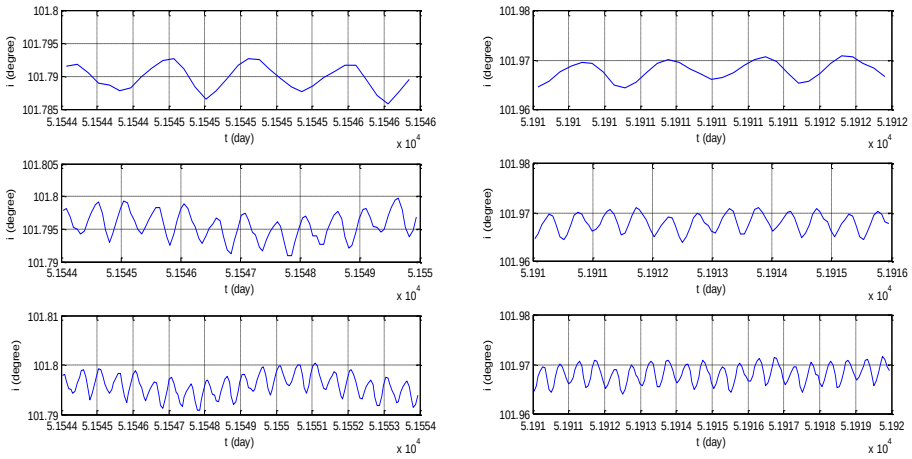


Figure 8-Change in inclination of NOAA 2 and NOAA 3 for 2, 6 and 10 days.

Figure-9 shows the perturbation effect on NOAA's satellites for long periods (1,5 and 10 years) where the results show that the variation in $(\Delta i/i)$ increases with increasing the period of the orbit. The detailed results are given in Table-7.

Table 7-Change in inclination of NOAA's satellites over a long period

Satellite	$i(degree)$	1 year $(\Delta i/i)$	5 year $(\Delta i/i)$	10 year $(\Delta i/i)$
NOAA 2	101.7882	0.0001474	0.0005896	0.0010807
NOAA 3	101.9573	0.0002256	0.0005884	0.0009808

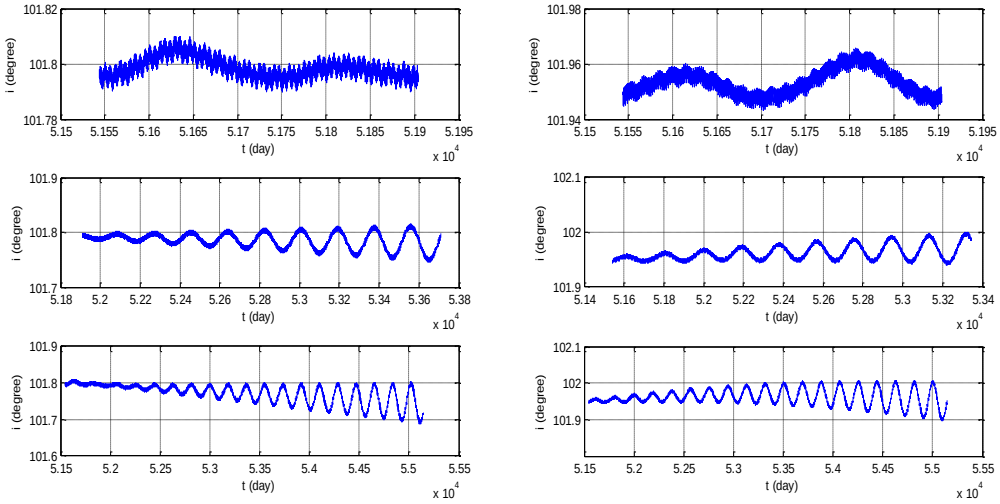


Figure 9-Change in inclination of NOAA 2 and NOAA 3 for 1, 5 and 10 years.

Also, Figure-10 shows that $(\Delta a/a)$ remains constant for NOAA's satellites over the long period revolution (Table-8).

Table 8-Perturbation on the semi-major axis of GEO satellites over long periods

Satellite	$a(Km)$	1 year $(\Delta a/a)$	5 year $(\Delta a/a)$	10 year $(\Delta a/a)$
NOAA 2	6450	0.00001378	0.00001378	0.00001378
NOAA 3	6500	0.00000342	0.00000342	0.00000342

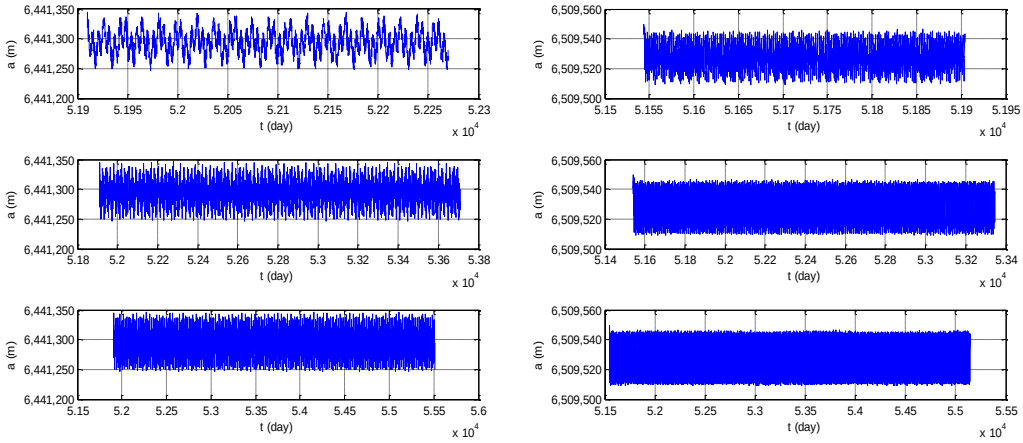


Figure 10-Perturbation on semi-major axis of NOAA 2 and NOAA 3 for 1, 5 and 10 years.

Conclusions

The earth tides cause perturbations in the motion of the close earth satellite. Variations in the inclination of GEO and NOAA satellites were computed. The results show that the variation in $(\Delta i/i)$ decreases with increasing the inclination of the satellite. We conclude that the tidal perturbation on the orbital elements depends on the inclination of the satellite. We also found that the variation in $(\Delta i/i)$ increases with increasing the time.

References

1. Vallado D. A. **1997**. *Fundamental of Astrodynamics and Applications*. The McGraw-Hill Companies, Inc. College Custom Series.
2. Vanicek P. and Krakiwsky E. J. **1986**. *Geodesy the concepts*. Elsevier science publishers.
3. Goring D. G. and Walters R. A. **2002**. Ocean-tide loading and Earth tides around New Zealand. *New Zealand Journal of Marine and Freshwater Research*, **36**: 299-309.
4. Kozai Y. **1965**. Effect of the Tidal Deformation of the Earth on the Motion of Close Earth Satellites. *Tokyo Astronomical Observatory*, **17**(4): 395-402.
5. Cazenave A., DailletS. and Lambeck K. **1977**. Tidal studies from the perturbations in satellite orbits. *Phil. Trans: R. Soc. Lond.* A.**284**: 595-606.
6. Lambeck K. **1974**. Solid Earth and Ocean Tides Estimated From Satellite Orbit Analyses. *REVIEWS OF GEOPHYSICS AND SPACE PHYSICS*. **12**(3): 422-434.
7. Seeber G. **2003**. *Satellite Geodesy. Walter de Gruyter Berlin New York* .
8. Mervart L. and Verdun A. **2005**. *Methods of Celestial Mechanics*, Volume I. Springer-Verlag Berlin *Heidelberg*
9. Kaula,W. M. **1966**. *Theory of satellite geodesy*. Blaisdell Publ. Comp., Waltham, Mass., London
10. Brouwer, D., Clemence, G. **1961**: *Methods of Celestial Mechanics*. Academic Press, New York, London.