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Finding the Best Technique to Transfer from Geosynchronous Transfer Orbit (GTO) to Lunar Orbit

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Abstract

To assess various methods for relocating a fictional satellite from a geosynchronous transfer orbit (GTO) to a lunar orbit, this research project was conducted. The primary objective of the study was to determine the most time and cost-effective technique, while also considering the influence of perturbations and other possible variables on orbital mechanics analysis. Three different approaches were examined, involving the transfer of the satellite in one, two, or three separate rounds. The development of algorithms for this investigation relied on MATLAB orbital mechanics software, and careful consideration was given to factors such as delta-v, mission duration, spacecraft mass, and potential perturbations throughout the course of the transfer.

For each mission's requirements, the study discovered that every technique had its own set of benefits and drawbacks. The least time-consuming and the easiest way was the first technique, despite using up the largest amount of propellant. The second technique might find a middle ground between propellant usage and mission time. Even though it took longer, the third technique consumed less propellant than the first two techniques. The fourth technique proved advantageous in terms of propellant usage and mission time. Factors such as atmospheric drag, perturbations from other celestial bodies, and solar radiation pressure can affect the spacecraft's trajectory and require additional analysis to ensure the success of the mission. The study also emphasized the impact of these perturbations on the spacecraft's path, potentially necessitating course corrections and increasing propellant usage. The most pronounced effects on the orbital elements were discovered to stem from the Earth's oblateness, primarily impacting perigee and apogee.

Providing insights into various mission requirements and perturbations, this study revealed the most efficient technique for transferring a satellite from the geosynchronous transfer orbit (GTO) to the lunar orbit.

Keywords: GTO; satellite; techniques; orbit elements; perturbations; lunar orbit

إيجاد أفضل تقنية للنقل من المدار الأرضي المتزامن (GTO) إلى مدار قمري

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الخلاصه

لتقييم الطرق المختلفة لنقل قمر صناعي افتراضي من مدار متزامن مع الأرض إلى مدار قمري، تم إجراء هذا البحث. كان الهدف الأساسي من الدراسة هو تحديد التقنية الأكثر فعالية من حيث الوقت والتكلفة، مع

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مراعاة تأثير الاضطرابات والمتغيرات الأخرى المحتملة على تحليل ميكانيكا المدار. تم فحص ثلاث طرق مختلفة، والتي تضم نقل القمر الصناعي في مسار واحدة أو مسارين أو ثلاث مسارات منفصلة. اعتمد تطوير الخوارزميات لهذا التحقيق على برنامج MATLAB للميكانيكا المدارية، وتم النظر بعناية في عوامل مثل دلتا-V، ومدة المهمة، وكتلة المركبة الفضائية، والاضطرابات المحتملة خلال عملية النقل .بالنسبة لمتطلبات كل مهمة، اكتشفت الدراسة أن كل طريقة لها مجموعة من المزايا والعيوب الخاصة بها. الطريقة الأسهل والأقل استهلاكا للوقت هي التقنية الأولى، على الرغم من أنها تستخدم أكبر كمية من الوقود الدافع. قد يجد التقنية الثانية حلاً وسطاً بين استخدام الوقود الدافع ومدة المهمة. على الرغم من أنها تستغرق وقتًا أطول، إلا أن التقنية الثالثة تستهلك وقودًا أقل من الأسلوبين الأولين. التقنية الرابعة مفيدة من حيث استخدام الوقود ومدة المهمة. يمكن أن تؤثر عوامل مثل السحب الجوي والاضطرابات من الأجرام السماوية الأخرى وضغط الإشعاع الشمسي على مسار المركبة الفضائية وتتطلب تحليلاً إضافيًا لضمان نجاح المهمة. تؤكد الدراسة أيضًا على تأثير هذه الاضطرابات على مسار المركبة الفضائية، مما قد يتطلب تصحيح المسار وزيادة استخدام الوقود. تم اكتشاف التأثيرات الأكثر وضوحًا على العناصر المدارية على أنها تنتج من انحراف الأرض، والتي تؤثر بشكل أساسي في الحضيض والأوج.

تقدم هذه الدراسة نظرة ثاقبة حول متطلبات المهمة والاضطرابات المختلفة، وتكشف عن أكثر التقنيات كفاءة لنقل القمر الصناعي من المدار المتزامن مع الأرض إلى المدار القمري.

1. Introduction

For centuries, humanity has been entranced by the vast and mysterious domain referred to as space, following the Earth's sprawling existence. While plumbing the depths of this realm, we have constantly been allured by the moon, which is situated close to our home planet. Our journey across the cosmos has provided us with a precious cache of information, including crucial data, precious samples, and priceless viewpoints. Of the lunar sphere and the celestial system at large, which have come to be better understood through this vast adventure [1, 2]. Our desire to understand the cosmos, solve universal mysteries, and comprehend the fundamentals of life spurs us forward. In fact, we may have made significant socio-economic gains and fueled high-tech industry growth thanks to new techniques and scientific advancements born from lunar exploration [3, 4]. Scientists gain a better understanding of the universe by observing celestial bodies, like the moon, and can explore various scientific questions [1]. The moon provides a fascinating source of much-needed resources that are difficult to procure on the Earth [5, 6]. Abundant compounds on the moon such as alumina, iron, titanium, magnesium, and silica suggest a generous supply of oxygen. Water is another essential resource located in deep craters near the poles of the moon [7, 8]. These icy water deposits have the potential to be valuable for future human colonies. Through nuclear fusion, the precious element helium 3, discovered in the moon, could be utilized as an efficient and clean energy source, according to researchers [9, 10, 11]. The moon's long day, stretching for two continuous weeks, also makes it an ideal location for solar energy generation [12, 13, 14]. The mission from the geosynchronous transfer orbit (GTO) to the lunar orbit can be accomplished in various ways; here we use three popular methods that stand out:

1. Direct transfer from the geosynchronous transfer orbit (GTO) to the lunar orbit. This method involves firing the spacecraft's engine once to leave GTO and head straight for the moon. It's fast but expensive, and the application of this method depends on the spacecraft's capabilities and launch conditions [15, 16].

2. The Hohmann transfer is a spacecraft maneuver that requires two separate engine firings [7, 8] Initially, the spacecraft fires its engine to depart from the geosynchronous transfer orbit (GTO) and enter a transfer orbit that intersects the Moon's orbit. Then another engine firing is performed to achieve a lunar orbit. If the initial, and final, or target orbits do not intersect, we need at least two momentums for inter-orbital transfer. This method is recognized for being highly fuel-efficient [17, 18].

3. This method involves three impulses; the bi-elliptic transfer requires the spacecraft's engine to be fired three times. The first firing is to exit the geosynchronous transfer orbit (GTO) and transition into a highly elliptical orbit encircling the Earth. To achieve its desired orbit, the spacecraft undergoes multiple engine burns. The first burn raises the spacecraft's apogee, then the second burn propels it to the same distance as the moon's orbit. Finally, a third burn allows it to enter the lunar orbit. This method, though more intricate and requiring additional engine burns, proves to be more efficient in terms of fuel consumption [19, 20].

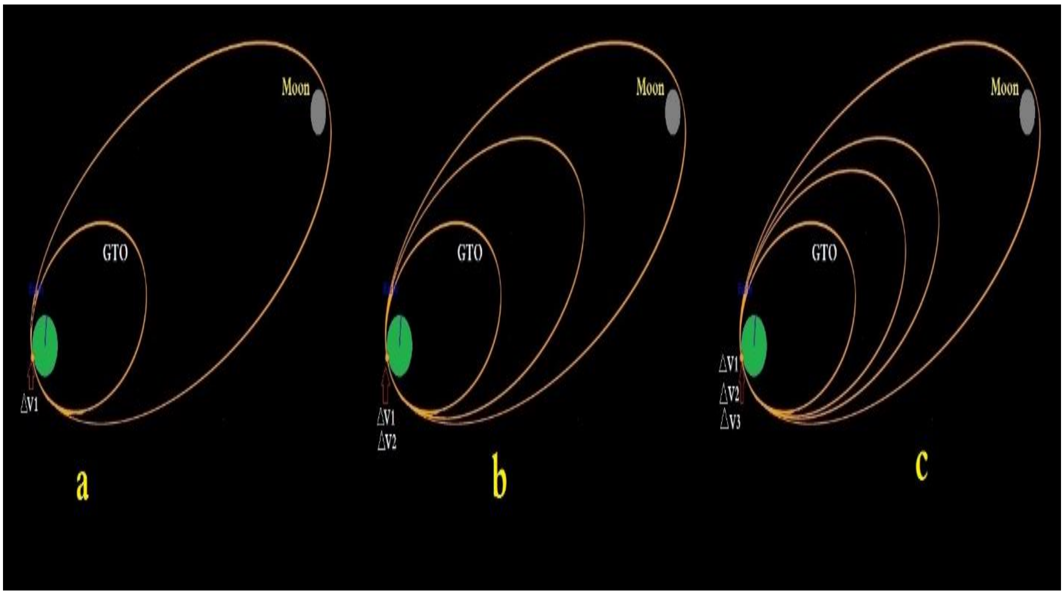

Figure 1: The diagram illustrates the ways to transfer a spacecraft from a geosynchronous transfer orbit (GTO) to the moon **(a)** Direct transfer **(b)** Hohmann transfer **(c)** Bi-elliptic transfer [19].

Describing the movement of a spacecraft typically involves employing the theoretical concept known as Keplerian motion, under the assumption that the spacecraft follows an elliptical course around a central mass. Nonetheless, a spacecraft's actual trajectory differs due to tiny disturbances caused by factors like the non-symmetrical shape of our planet, the gravitational pull exerted by other celestial bodies, and the drag experienced in the atmosphere. Considering the gravitational forces acting on the spacecraft, the perturbation equation can be used to describe the effects of these perturbations [21, 22].

Describing the motion of a spacecraft affected by small forces that alter its orbit from the predicted Keplerian path, the perturbation equation is expressed as follows [21, 22].

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2}\frac{r}{|r|} + A\,(r, v, t) \quad (1)$$

Where:

r: position vector of the celestial body.

G: gravitational constant.

M: mass of the attracting body.

A (r, v, t): perturbing forces.

|r|: magnitude of r.

This equation is solved by the orbital elements' calculation of position, velocity, and angular momentum:

1- Convert the position vector r and the velocity vector v to the orbital elements, which include the semi-major axis (a), the eccentricity (e), the inclination (i), the longitude of the ascending node (Ω), the argument of perigee (ω), and the true anomaly (f) [23, 24].

2- After getting the orbital elements, calculate the angular momentum from:

$$\mathbf{h} = \mathbf{r} \times \mathbf{v}. \tag{2}$$

Where ×: the cross product.

Then it can be rewritten as perturbation equation based on the orbital elements and the angular momentum from [23, 24]:

$$\frac{d^2}{dt^2}(\mathrm{a} \times \mathrm{r}) = -\frac{\mu}{a^2 \times \mathrm{r}} + \Delta\mathrm{a} \times \mathrm{r} + \mathrm{a} \times \Delta\mathrm{r} \tag{3}$$

Where:

μ: gravitational parameter.

Δa: change of the semi-major axis.

Δr: change of the position vector.

Because perturbations are complex and so large that a purely analytical solution to the perturbation equation is not tractable, we used numerical methods. Numerical methods, like the Runge-Kutta technique, can solve the orbital perturbation equation. This process begins by converting the position and velocity vectors to the orbital elements, then calculating the angular momentum, and, ultimately, writing the perturbation equation in terms of the orbital elements and the angular momentum. With these steps, the equation can be solved with easier and more reliable accuracy [25, 26, 27, 28].

Previous research and studies focused on two sections: the first was lunar mission strategies, and the second was the effects of disturbances on spacecraft paths. Rogan Shimmin (2013) studied the potential of using low-thrust propulsion for lunar missions by optimal trajectory design [29], while Richard Epenoy and Daniel Pérez-Palau (2019) designed low-energy transfer orbits between the geostationary orbit and the moon [30]. Similarly, Lorenzo Casalino and Gregory Lantoine (2020) analyzed lunar mission trajectories using the gravity assists of the Earth and the moon to reduce propellant use [31]. Also, in 2018, Mohammed A. Yousif and Abdul-Rahman H. Saleh study was concerned with the evaluation of orbital maneuvers for transitioning from low Earth's orbit to geostationary Earth's orbit using numerical simulations [18].

As for the perturbations, researchers have studied perturbations and their impact on mission trajectories, such as the study by Anas Salaman Taha (2002) which investigated disruptions of satellites, which was a disorder affecting the orbits of low-lying satellites [10]. Meanwhile, Prado (2002) investigated the disruptions in lunar satellites' orbits caused by the moon's gravitational field [32]. Al-Ali (2011) computed the perturbation effects on orbital elements of the moon, which computed the perturbations, including atmospheric drag, non-spherical earth, solar radiation pressure, and a third-body attraction. These perturbations disrupted an object's orbit and were also found to cause changes in the moon's orbital elements with time [33]. In 2016, Taif A. Damin and Abdulrahman H. Salih investigated the solar attraction effect on orbital elements of the moon [6]. Abdul-Rahman H. Salih et al. (2019) calculated the effects of the sun, the moon, and satellite position on the perturbation forces of the low retrograde orbits [34]. Meanwhile, Fouad M. Abdulla et al. (2016) described the orbital elements variation of the moon through 2000-2100 [35]. Finally, Farid M. Mahdi et al. (2020) studied the determination and evaluation of the orbital transition methods between two elliptical earth orbits [12]

2. Methods:

These equations are used in algorithms for the transfer methods used in this study [34, 36, 37]: -

$$\mathrm{a} = \frac{r_p + r_a}{2} \tag{4}$$

Where:

a: semi-major axis of the elliptical orbit.

r_p: perigee distance.

r_a: apogee distance.

$$n = \sqrt{\frac{\mu}{a^3}} \tag{5}$$

Where:
n: Mean motion.
μ: the gravitational parameter of the Earth.

$$T = \frac{2 \times \pi}{n} \tag{6}$$

T: period of the spacecraft.

$$e = \frac{(r_a - r_p)}{2\,a} \tag{7}$$

Where:
e: eccentricity.

$$V_p = \sqrt{\mu \frac{2}{r_p} - \frac{1}{a}} \tag{8}$$

Where:
V_p: velocity at perigee.

$$E = \frac{{V_p}^2}{2} - \frac{\mu}{r_p} \tag{9}$$

Where:
E: the energy of the body in its orbit.

$$V_a = \sqrt{\mu \frac{2}{r_a} - \frac{1}{a}} \tag{10}$$

Where:
V_a: the Velocity spacecraft at apogee.

$$\text{Delta V} = \sqrt{(2\,(E2 - E1))} \tag{11}$$

Where:
E2: the orbital energies of the final orbit.
E1: the orbital energies of the initial orbit.
The total change in orbital energy (ΔE_{total}) can be calculated as follows:

$$\Delta E_{total} = -\frac{\mu}{2 \times a_p} \times \left(\sqrt{\frac{2a_a}{a_p + a_a}} - 1\right)^2 \tag{12}$$

Where:
a_p: semi-major axis of the initial orbit.
a_a: semi-major axis of the final orbit.
$\left(\sqrt{\frac{2a_a}{a_p + a_a}} - 1\right)$: the ratio of the velocity at apogee of the final orbit to the velocity at perigee of the initial orbit [6, 31, 33, 37, 38, 39]
A negative sign: the burn is retrograde [40, 41, 42, 43, 44, 45]

The total change in specific orbital energy for a Hohmann transfer at perigee between two elliptical orbits with the same perigee and different apogee, where the change in specific energy during each burn is calculated using the velocity at the transfer perigee and the transfer perigee radius. The total change in specific orbital energy is then the sum of the specific energy changes during each burn [40, 46]:

- The change in energy at the first burn can be calculated [40, 46]:

$$\Delta E_1 = \frac{{V_{p1}}^2}{2} - \frac{\mu}{r_p} \tag{13}$$

Where:
v_{p1}: the velocity at the first transfer perigee.
r_p : the first transfer perigee radius.

The change in energy at the second burn can be calculated [40, 47]:

$$\Delta E_2 = \frac{V_{p2}^{\;2}}{2} - \frac{\mu}{r_p} \tag{14}$$

Where:
$v_{p2}^{\;2:}$ velocity at second transfer perigee.
r_p : second transfer perigee radius.
The total change in orbital energy (ΔE_{total}) given by [40, 48]:

$$\Delta E_{total} = \Delta E_1 + \Delta E_2 \tag{15}$$

The total change in orbital energy for a bi-elliptical transition at the perigee can be calculated using formulas [33, 40, 49]:

$$\Delta E_{total} = \Delta E_1 + \Delta E_2 + \Delta E_3 \tag{16}$$

$$\Delta E_1 = \frac{V_{p2}^{\;2}}{2} - \frac{\mu}{r_p} \tag{17}$$

$$E_2 = \frac{V_{p3}^{\;2}}{2} - \frac{\mu}{r_p} \tag{18}$$

$$\Delta E_3 = \frac{V_{p4}^{\;2}}{2} - \frac{\mu}{r_p} \tag{19}$$

3. Results and Discussion

According to previous space missions, there are known and approved methods for transporting any satellite or spacecraft to outer space, specifically the moon. Therefore, in this study, we developed four techniques that help reduce the time and energy consumed for such trips. These techniques use different internationally approved methods to transport a spacecraft or satellite into space, where the first technique uses direct transfer, while the second and third techniques use Hohmann transfer. Finally, the fourth technique uses bi-elliptical transfer. In this study, we will analyze each technique by showing its negatives and positives, and compare them to achieve the goal of this study, which is to obtain the best technique to transfer a virtual satellite from a geosynchronous transition orbit (GTO) to a lunar orbit with the least time and energy consumption, then study the effect of perturbations on these techniques to determine the conditions of Truth facing satellite mission.

The results and suggestions obtained from this study will facilitate future space missions, as it examines the most critical problem facing space mission designers, which is the long mission time, in addition to the high cost due to the energy consumed, as follows:

- The first technique:

The direct transfer method, in which the transmission takes place at one time from the primary orbit to the target orbit. Accordingly, in the first technique that uses this method, the satellite will be moved from the geosynchronous transitional orbit (GTO) (primary orbit) to the lunar orbit (target orbit). The results (Table 1 and Figure 2 (a)) of this technique show the following:
- The least time it takes to move to lunar orbit.
- The highest change of velocity (delta-V) corresponds to high energy consumption (delta-E).

Based on the above information, this technique has succeeded in solving the time problem, but it will require high energy consumption to transfer the satellite to the orbit of the moon. Therefore, the use of this technique will be very limited to missions whose most important condition is time, and fuel consumption does not matter.
Advantages of this technique: little time is taken compared to other techniques.
Disadvantages of this technique: high energy consumption, which is not limited to the mission path, but this technique will need additional modifications that require higher fuel consumption to overcome the perturbations that may be encountered for the satellite mission.

Table 1: Delta-V, delta-E, and the time for the first technique.

	LEO to GTO		**GTO to Lunar orbit**				
The 1st technique	**V_{p1} (km/sec)**	**V_{a1} (km/sec)**	**V_{p2} (km/sec)**	**V_{a2} (km/sec)**	**V_{pf} (km/sec)**	**V_{af} (km/sec)**	**ΔV_{total} (km/sec)**
	7.81822802	**1.597935**	**10.716236**	**1.3930569**	**10.626822**	**0.3499362**	**0.089414**
Energy (J/kg)	**-29.5124621**	**-8.14718789**	**-0.0000012**	**-0.0000012**	**-0.9977647**	**-0.9977647**	**-0.91329300**
The time of mission (day)	**0.06398785**		**10.28300**				
The time of mission total (day) is 10.346991							

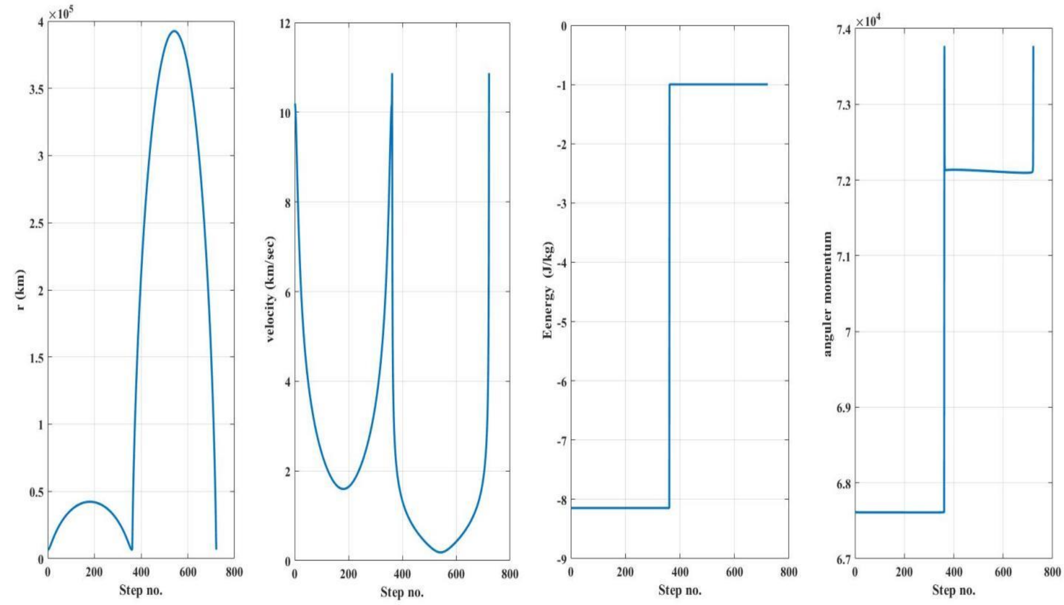

Figure 2 (a) :Changes in distance, velocity, and energy, as well as angular momentum, are specific to the first technique.

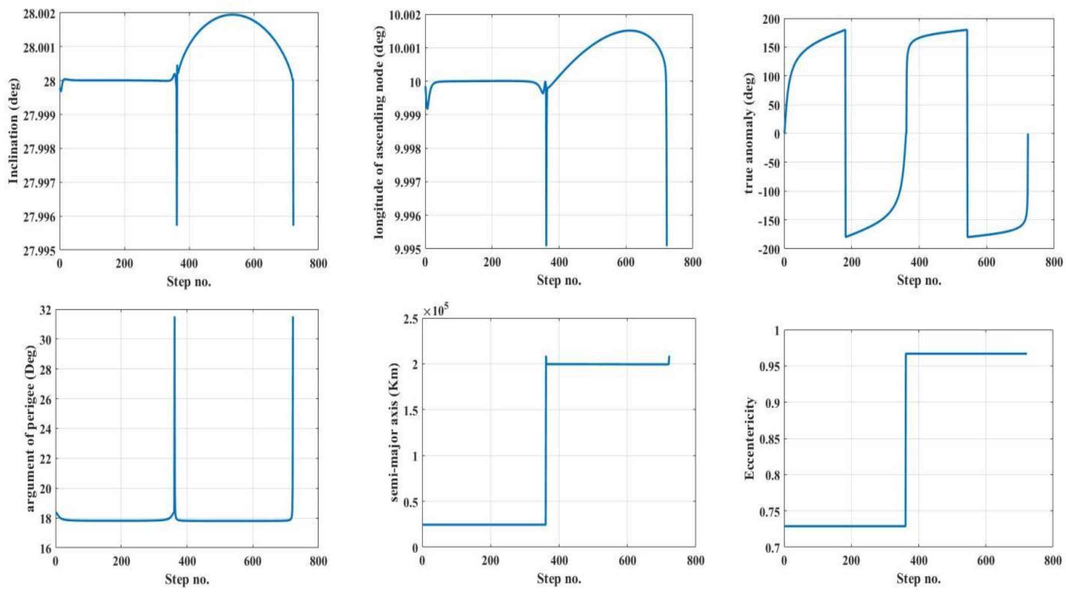

Figure 2: (b) The orbital elements change with perturbations for the first technique.

- The second technique:

As for the second technique, which uses the Hohmann transfer method, that is, the transmission takes place twice: from the primary orbit to a chosen altitude and then from the chosen altitude to the target orbit. The mechanism adopted in this technique is that, first, the satellite is launched from the geosynchronous transitional orbit GTO (primary orbit) to an altitude of 120,000 km. Secondly, a second push is made to move from the height of 120,000 km to the lunar orbit (target orbit). The results are given in Table 2 and Figure 3(a): -

- More transmission time than the first technique by a small rate.
- The values of change in velocity delta-V are higher than the first technique, and therefore the delta-E values are lower (the amount of fuel consumption is less), but it is not less than the other two techniques.

Based on this information, the advantages of this technique are that the amount of fuel consumption is less than that of the first technology, but it will also require fuel consumption to escape from the gravity of the Earth at an altitude of 120,000 km, which is considered one of its defects, as well as other defects in requiring more time to move to the required orbit, which may not constitute a serious problem in space missions.

Table 2: Delta-V, delta-E, and the time for the second technique.

	LEO to GTO		GTO to Lunar orbit								
	V_{p1} (km/sec)	V_{a1} (km/sec)	V_{p2} (km/sec)	V_{a2} (km/sec)	V_{p3} (km/sec)	V_{a3} (km/sec)	V_{pf} (km/sec)	V_{af} (km/sec)	ΔV_1 (km/sec)	ΔV_2 (km/sec)	ΔV_{total} (km/sec)
The 2nd technique	**7.81822802**	**1.597935**	**10.7084022**	**1.3932123**	**10.618905**	**2.0974678**	**10.614820**	**0.18798986**	**0.089497**	**0.004085**	**0.0935822**
Energy (J/kg)	**-29.5124621**	**-8.14718789**	**-0.00000239**	**-0.00000239**	**-0.954368**	**-0.954368**	**-0.99774**	**-0.99774**	**-0.954366**	**-0.997659**	**-1.9521071**
The time of mission (day)	**0.06398785**		**12.26603310**								
The time of mission total (day) is 12.33002095											

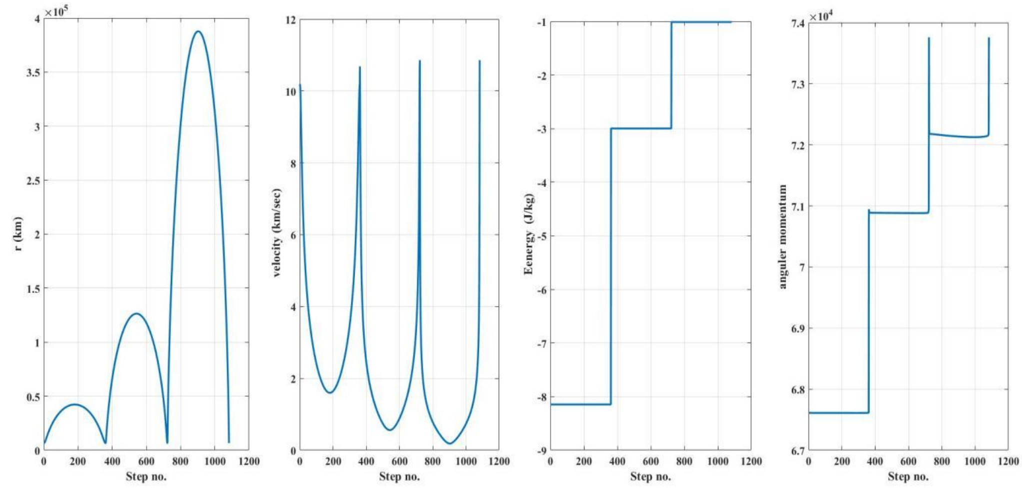

Figure 3: (a) Changes in distance, velocity, and energy, as well as angular momentum, are specific to the second technique.

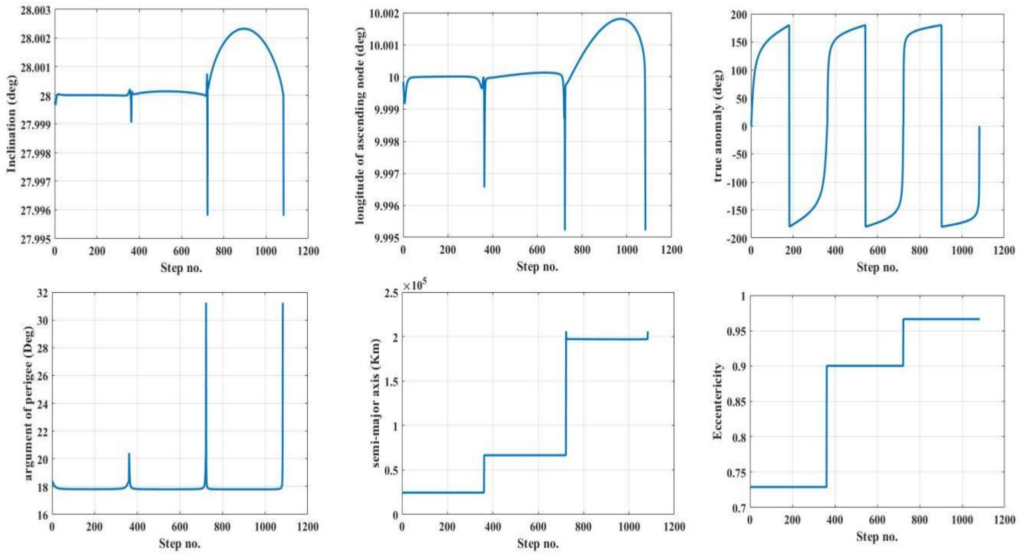

Figure 3: (b) The orbital elements change with perturbations for the second technique.

- The third technique:

The third technique uses the same Hohmann transfer as the second technique, and it also follows the same mechanism used by the second technique, but the chosen height is different, as it has used in this technique a height of 200,000 km. I.e., the transmission is done first from the geosynchronous transitional orbit GTO (primary orbit) to 200,000 km, then a second burn is made to move from the height of 200,000 km to the lunar orbit (target orbit). The results, shown in Table 3 and Figure 4-a, gave:

- Longer transmission time compared to the first and second techniques, but not longer than the fourth technique.
- the values of velocity change are less delta-V than the first and second techniques, and therefore the amount of fuel consumption is less delta-E, than the first and second techniques, but it is not less than the fourth technique.

So, the advantage of this technique is that the amount of fuel consumed is less than the first and second techniques, but its disadvantage is that it will take longer than the first and second techniques, although, in space missions, time does not constitute a real problem [50].

Table 3 : Delta-V, delta-E, and the time for the third technique.

The 3rd technique	LEO to GTO		GTO to Lunar orbit								
	V_{p1} (km/sec)	V_{a1} (km/sec)	V_{p2} (km/sec)	V_{a2} (km/sec)	V_{p3} (km/sec)	V_{a3} (km/sec)	V_{pf} (km/sec)	V_{af} (km/sec)	ΔV_1 (km/sec)	ΔV_2 (km/sec)	ΔV_{total} (km/sec)
	7.81822802	1.597935	10.6839389	1.3936929	10.5941810	1.3974835	10.5901481	0.1884128	0.089758	0.004033	1.6742757
Energy (J/kg)	-29.5124621	-8.14718789	-0.00000239 57	-0.00000239 57	-0.954943	-0.954943	-0.997660	-0.997660	-0.95494251	-0.99765968	-1.9526022
The time of mission (day)	0.06398785		14.576420489								
The time of mission total (day) is 14.64040833											

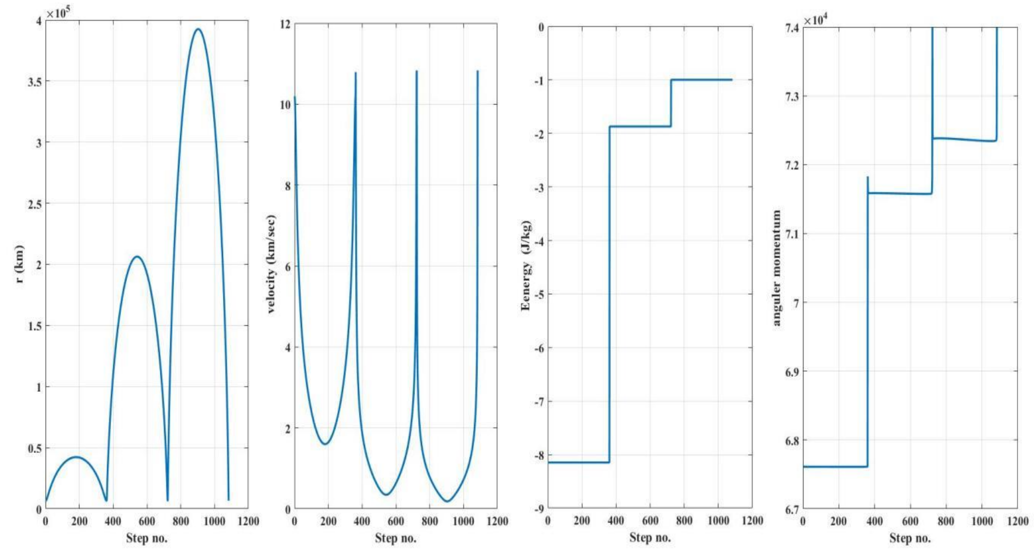

Figure 4: (a) Changes in distance, velocity, and energy, as well as angular momentum, are specific to the third technique.

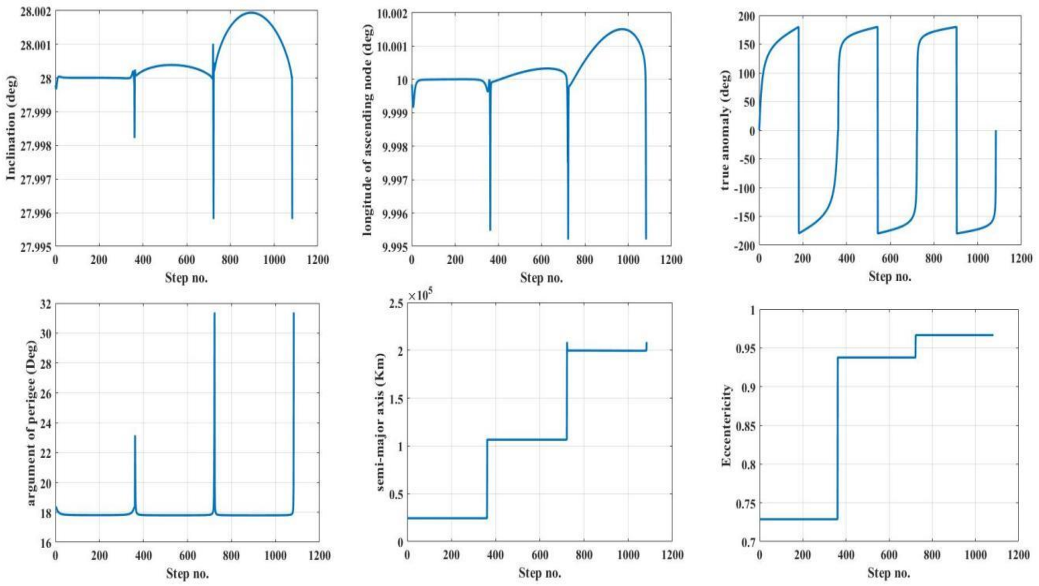

Figure 4: (b) The orbital elements change with perturbations for the third technique.

- The fourth technique:

The fourth and final technique uses the bi-elliptical transfer method; that is, the transmission takes place in three stages. The mechanism adopted in this technique is: firstly, a transition is made from the primary orbit to a chosen height. Secondly, a transition is made from the chosen height to a second chosen height. Finally, a transition is made from the second chosen height to the target orbit. Accordingly, it is done as follows: first, a first burning is performed, and the transition from the geosynchronous transitional orbit GTO (primary orbit) to an altitude of 120,000 km, then, a second burning is performed, and the transition is made from an altitude of 120,000 km to a height of 2,800 km. Finally, the third and final burn is made to move from the altitude of 280,000 km to the lunar orbit (target orbit), and according to Table 4 and Figure 5-a, the results were:

- Longer transmission time than the first, second, and third techniques.
- The values of change in velocity delta-V are less than the first, second, and third techniques, and therefore the amount of fuel consumption is less delta-E than the first, second, and third techniques.

Accordingly, the advantage of this technique is that the amount of consumed fuel that is less than the other three techniques chosen in the study, and its disadvantage is that the mission time will be longer compared to the rest of the techniques. As we mentioned earlier, time in space exploration flights does not constitute a fundamental problem.

Table 4 : Delta-V, delta-E, and the time for the fourth technique.

	LEO to GTO		GTO to Lunar orbit											
The 4th technique	**V_{p1} (km/sec)**	**V_{a1} (km/sec)**	**V_{p2} (km/sec)**	**V_{a2} (km/sec)**	**V_{p3} (km/sec)**	**V_{a3} (km/sec)**	**V_{p4} (km/sec)**	**V_{a4} (km/sec)**	**V_{pf} (km/sec)**	**V_{af} (km/sec)**	**ΔV_1 (km/sec)**	**ΔV_2 (km/sec)**	**ΔV_3 (km/sec)**	**ΔV_{total} (km/sec)**
	7.81822802	**1.5979935**	**10.6008525**	**1.3674525**	**10.5137411**	**2.1139493**	**10.4719441**	**0.2594041**	**10.5095610**	**0.1833763**	**0.0871113**	**0.0417971**	**0.0376170**	**0.1665254**
Energy (J/kg)	**-29.5124621**	**-8.1471878 9**	**-0.000002307**	**-0.00000231**	**-0.919662616**	**-0.91966262**	**-1.358232655**	**-1.35823266**	**-0.963602346**	**-0.96360235**	**-0.9196603**	**-1.3582327**	**-0.9636024**	**-3.24149761**
The time of mission (day)	**0.06398785**		**19.00758282**											
The time of mission total (day) is 19.07157067														

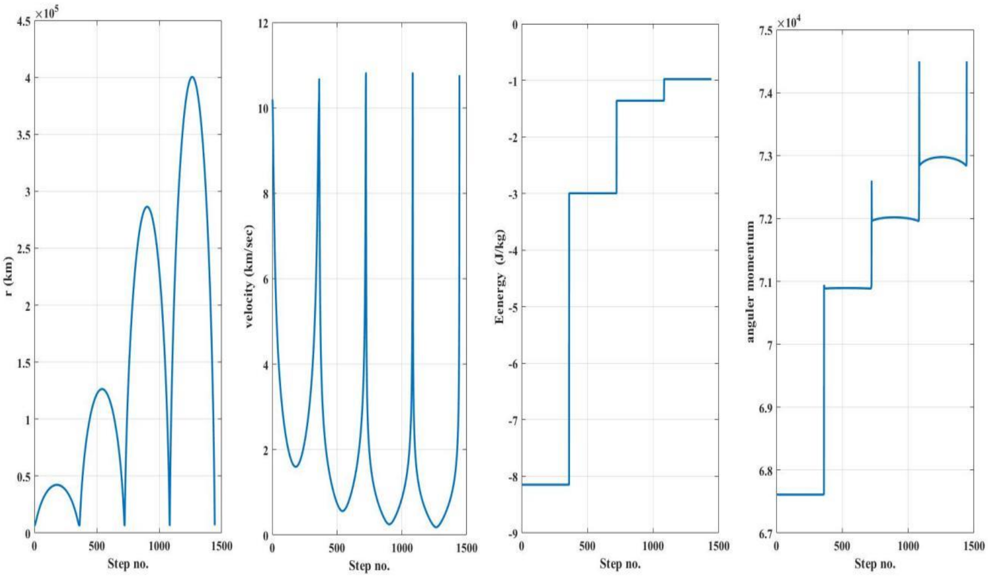

Figure 5: (a) Changes in distance, velocity, and energy, as well as angular momentum, are specific to the fourth technique.

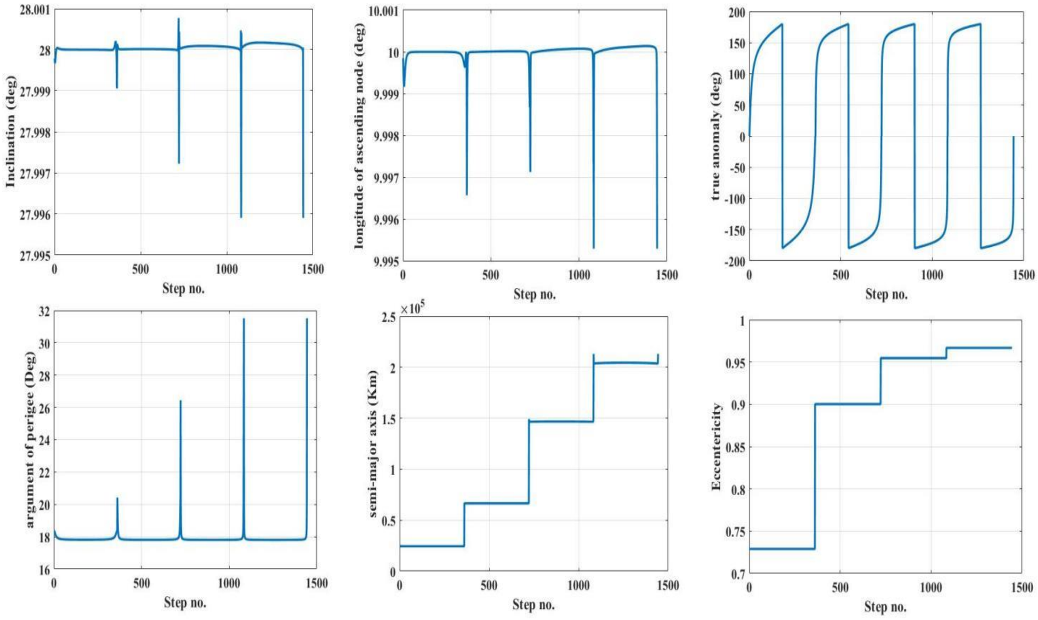

Figure 5: (b) The orbital elements change with perturbations for the fourth technique.

After analyzing the four techniques, identifying the advantages and disadvantages of each technique, and comparing them, we must note that all four techniques selected in this study were analyzed according to ideal conditions, which do not represent the real situation of the mission. Space missions in general, during the transition from the Earth to space, face a group of perturbations that may affect the space mission plan clipart, represented by atmospheric drag, non-spherical Earth, and the effect of the third body, not to mention the effect of solar radiation. Therefore, the impact of these perturbations on the satellite transition path will be studied for the selected techniques in this study to provide a picture closer to the real conditions for space missions.
Figures (1-b), (2-b), (3-b), and (4-b) show how perturbations may affect the orbital elements of the satellite during the transition for each technique. The idea of perturbations is very complex, as it can be affected by a number of diverse variables, where it is noted that most of the perturbations occur in the perigee and apogee regions; alternately, this is expected where the impact is due to Earth's oblateness. The perigee is the point closest to the Earth, so when the satellite is at this point, the effect of the Earth's gravity will be stronger, and with the presence of other perturbations at this point, the effect will be stronger (that is, all perturbations will be combined in this region). While apogee is the point farthest from the Earth, and therefore, when the satellite is at this point, the effect of gravity is weaker, and the effect of the rest of the perturbations (atmospheric drag and solar radiation pressure) is weaker in these areas, for this reason, perturbations are evident in these two points, alternately.

According to the above, the fourth technique is the least fuel-consuming, which makes it the most efficient, followed by the second and third techniques, which are considered the middle ground between the four techniques. As for the first technique, although it consumes more fuel and is less efficient, it can be used on some missions where time is the only condition in the mission.

Conclusion

From this study, it can be concluded that the best technique that should be adopted is the fourth technique because it solves both time and fuel consumption issues compared to other technologies. If the satellites are transported from the Geostationary Transfer Orbit (GTO) to the lunar orbit in three batches, more efficient and precise mission control can be achieved in no more than 19 days. Therefore, it is deemed the most suitable and successful method for achieving the study's objective.

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