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Electric Quadrupole Transitions of Some Even-Even Neon Isotopes

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Abstract:

The reduced electric quadrupole transition strengths B(E2) from the first excited 2^+ state to the ground 0^+ state of some even-even Neon isotopes ($^{18,20,22,24,26,28}Ne$) have been calculated. All studied isotopes composed of ^{16}O nucleus that is considered as an inert core and the other valence particles considered to move over the sd-shell model space within $1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$ orbits.

The configuration mixing shell model with limiting number of orbitals in the model space outside the inert core fail to reproduce the measured electric transition strengths. Therefore, and for the purpose of enhancing the calculations, the discarded space has been included through a microscopic theory which considers a particle-hole excitations from the core orbits and from the model space orbits into the higher orbits with $2\hbar\omega$ excitations. These effects are called core-polarization effects.

The transition strengths have been calculated with effective nucleon charges deduced from the core polarization calculations, and calculated with standard effective nucleon charges $e_p^{eff} = 1.3e$ for the proton and $e_n^{eff} = 0.5e$ for neutron. The calculations are based on sd-shell model space with USDB interactions.

The harmonic oscillator potential is used to generate the single particle matrix elements, where the value of the size parameter *b* is adjusted to get the experimental root mean square matter radii for each isotope.

Keywords: electric quadrupole transition, transition strength, even-even Neon isotopes.

الانتقالات الكهربائية رباعية القطب لبعض نظائر النيون الزوجية – زوجية

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الخلاصة:

تم حساب قوى الانتقالات الكهربائية رباعية الاقطاب المختزلة B(E2) من المستوي المتهيج الاول 2^+ الى المستوي الأرضي 0^+ لبعض نظائر النيون الزوجية – زوجية ($^{18,20,22,24,26,28}Ne$). ان جميع النظائر المدروسة تتشكل من نواة ^{16}O والتي افترض على انها قلب خامل اضافة الى جسيمات خارج هذا القلب افترض بأنها تتحرك ضمن أنموذج فضاء القشرة sd وفي المدارات $1d_{5/2}$، $2s_{1/2}$ و $1d_{3/2}$.

ان أنموذج القشرة مختلط التشكيلات والذي يحتوي على عدد محدد من المدارات في أنموذج الفضاء خارج القلب الخامل يفشل في تفسير نتائج قوى الانتقالات الكهربائية المقاسة عمليا. لذلك ولغرض تحسين الحسابات، تم تضمين الفضاء المستثنى والذي يضم مدارات القلب الخامل والمدارات العليا في نظرية مجهرية تفترض استثارة زوج (جسيم - فجوة) من مدارات القلب ومدارات أنموذج الفضاء بطاقة استثارة مقدارها حيث تدعى هذه العملية (تأثير استقطاب القلب).

ان قوى الانتقالات المختزلة تم حسابها بدلالة شحنات النيوكليون الفعالة والمستنبطة من حسابات استقطاب القلب، كما وتم حسابها بدلالة شحنات النيوكليون القياسية والتي مقدارها للبروتون و $e_n^{eff} = 0.5e$ للنيوترون. ان الحسابات تم اجراءها ضمن أنموذج فضاء القشرة sd وبتفاعل USDB.

استخدم جهد المتذبذب التوافقي في توليد عناصر المصفوفة للجسيمة المفردة حيث افترضت قيمة مناسبة للثابت التوافقي b لكل نظير وذلك لإعادة توليد معدل الجذر التربيعي لإنصاف الاقطار المادية أو الكتلية.

Introduction:

The study of the properties of nuclei far from the stability valley is one of the major topics of interest of modern nuclear physics. Recent advancements in experimental techniques have probed extreme types of nuclear structures not previously known [1], termed 'exotic nuclei' which are nuclei with extreme properties, such as an extraordinary ratio of protons and neutrons Z/N, loosely binding energy, short-lived isotopes and large isospin. These unstable nuclei (far from the betastability line) are generally weakly bound with few excited states and have a thin cloud of nucleons orbiting at large distances from the others that forming an inert core. With the development of the on-line mass separators coupled to powerful accelerators, a wide range of short-lived nuclei far off stability became accessible for investigation. But most of these nuclei are available in trace quantities only and for a limited amount of time. Hence the usual scattering and nuclear reaction experiment using these nuclei as target or beam is difficult to perform. The low intensities of radioactive ion beams available at present coupled with their short lifetimes require special measurement techniques to be employed for exotic nuclei [2]. To learn about correlations away from beta-stability, one often needs to do experiments with short-lived radioactive nuclei. Since one cannot make a target from short-lived nuclei,these experiments are done in inverse kinematics. This means that rather than a beam of e.g. electrons being shot at a stationary target to knock out a nucleon from it, the beam itself contains the nucleus of interest and is shot at a hadronic target (usually a light nucleus such as ^9Be or ^{12}C) which induces the knockout. The first series of experiments with unstable nuclear beams aimed at determining nuclear sizes by measuring the interaction cross section σ_I in high energy collisions was performed by Tanihata and coworkers in 1985 [3,4]. The σ_I was measured with transmission-type experiments. Successive use of this technique has yielded nuclear size data over a wide range of isotopes where the measured cross sections have been used to deduce the root mean square matter (rms) radii by a Glauber-model analysis in the optical limit [5]. The growth of a neutron skin with the neutron number in several isotopes has been deduced from nuclear and charge-size data. This type of experiment has been continued at the Fragment Separator (FRS) and there exists an extensive quantity of measured interaction and reaction cross-sections for isotopes ranging from ^3He to ^{32}Mg [6].Other techniques, e.g. isotope-shift measurements, have allowed to extract the charge size.

The sd shell nuclei are very interesting from a theoretical point of view, because they are very appropriate for a study of the applicability of nuclear models, the property of the residual interactions and the mechanism of nuclear rotational motion. Among a number of models, the nuclear shell model is one of the most successful, where residual forces play a very important role. These interactions can induce configuration mixings, which can sometimes be interpreted as rotational motion.A study of nuclei in the sd shell can thus lead to a better understanding between a microscopic description of the nucleus (shell model) and a macroscopic (collective) description [7]. The calculations of shell model, carried out within a model space in which the nucleons are restricted to occupy a few orbits are unable to reproduce the measured static moments or transition strengths without scaling factors. Shell model within a restricted model space is one of the models, which succeeded in describing static properties of nuclei, when effective nucleon charges are used. Calculations of transition strengths using the model space wave function alone are inadequate for reproducing the data. Therefore, effects out of the model space, which are called core polarization effects, are necessary to be included in the calculations.

The sd-shell nuclei are considered as an inert ^{16}O core and the valence nucleons are distributed over $1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$ states. Higher configurations can be included through perturbation theory, where particle-hole excitations are allowed from the core and the valence nucleons to all allowed orbits with $n\hbar\omega$ excitations. The number n depends on the convergence of the calculations. These effects are called core-polarization effects. Anyway, the core-polarization effects on the electromagnetic transition strength are based on a microscopic theory, which combines shell model wave functions and configurations with higher energy as first order perturbation.

In the present work, the rms matter radius is reproduced by fixing the size parameter of the single particle wave function of the harmonic oscillator (HO) potential for each isotope in its ground state, where ^{16}O nucleus considered as an inert core. The reduced electric quadrupole transition strengths B(E2) from the first excited 2^+ state to the ground 0^+ state of some even-even Neon isotopes ($^{18,20,22,24,26,28}Ne$) are calculated by using the effective nucleon charges which deduced from the core polarization calculations with excitations up to $2\hbar\omega$, and calculated by using the standard effective nucleon charges $e_p^{eff} = 1.3e$ and $e_n^{eff} = 0.5e$ for protons and neutrons, respectively; and the results are compared with the available experimental values. The occupation numbers of the valence particles outside ^{16}O core are also calculated for the ground state of $1d_{3/2}$, $1d_{5/2}$ and $2s_{1/2}$ orbits to identify the contribution of these orbits for the valence particles.

Theory:

The longitudinal (Coulomb) one-body operator for a nucleus with multipolarity J and momentum transfer q is given by [8]:

$$\hat{T}_{JM}(q) = \int d\vec{r} j_J(qr) Y_{JM}(\Omega) \hat{\rho}(\vec{r}) \qquad (1)$$

where $j_J(qr)$ is the spherical Bessel's function, $Y_{JM}(\Omega)$ is the spherical harmonics and $\hat{\rho}(\vec{r})$ is the density operator, which is given by:

$$\rho(\vec{r}) = \sum_k e(k) \delta(\vec{r} - \vec{r}_k) \qquad (2)$$

The longitudinal (Coulomb) one-body operator becomes:

$$\hat{T}_{JM}(q,\vec{r}) = \sum_{k=1}^{n} e(k)\, j_J(qr)_k Y_{JM}(\Omega_k) \qquad (3)$$

where $e(k)$ is the electric charge for the k-th nucleon. Since $e(k)$= 0 for neutron, there should appear no direct contribution from neutrons; however, this point requires further attention: The addition of a valence neutron will induce polarization of the core into configurations outside the adopted model space. Such core polarization effect is included through perturbation theory which gives effective charges for the proton and neutron. Equation (3) can be written as:

$$\hat{T}_{JM}(q,\vec{r}) = \sum_{k=1}^{n}\left(e_{\mathrm{p}}\frac{1+\tau_z(k)}{2} + e_{\mathrm{n}}\frac{1-\tau_z(k)}{2}\right) j_J(qr)_k Y_{JM}(\Omega_k) \quad (4)$$

where $\tau_z|\mathrm{p}\rangle = |\mathrm{p}\rangle$ and $\tau_z|\mathrm{n}\rangle = -|\mathrm{n}\rangle$. Equation (4) can be rearranged to:

$$\hat{T}_{JM}(q,\vec{r}) = \sum_{k=1}^{n}\left(\frac{e_{\mathrm{p}}+e_{\mathrm{n}}}{2} + \frac{e_{\mathrm{p}}-e_{\mathrm{n}}}{2}\tau_z(k)\right) j_J(qr)_k Y_{JM}(\Omega_k) \quad (5)$$

which can be written as:

$$\hat{T}_{JM}(q,\vec{r}) = e_{\mathrm{IS}}\sum_{k=1}^{n} j_j(qr)_k Y_{JM}(\Omega_k) + e_{\mathrm{IV}}\sum_{k=1}^{n} j_j(qr)_k Y_{JM}(\Omega_k)\tau_z(k) \quad (6)$$

where $e_{\mathrm{IS}} = \frac{e_{\mathrm{p}}+e_{\mathrm{n}}}{2}$ and $e_{\mathrm{IV}} = \frac{e_{\mathrm{p}}-e_{\mathrm{n}}}{2}$ (7)

are the isoscalar and isovector charges, respectively. The bare proton and neutron charges are denoted by e_{p} and e_{n} , respectively.

The reduced matrix element in both spin-isospin spaces of the longitudinal operator $\hat{T}_\Lambda$ is expressed as the sum of the product of the elements of the one-body density matrix (OBDM) $X^{\Lambda}_{\Gamma_f\Gamma_i}(\alpha,\beta)$ times the single-particle matrix elements, and is given by [9]:

$$\left\langle \Gamma_f \,|||\, \tilde{\hat{T}}_\Lambda \,|||\, \Gamma_i \right\rangle = \sum_{\alpha\beta} X^{\Lambda}_{\Gamma_f\Gamma_i}(\alpha,\beta)\left\langle \alpha \,|||\, \tilde{\hat{T}}_\Lambda \,|||\, \beta \right\rangle \quad (8)$$

where α and β label single-particle states (isospin is included) for the shell model space. The states $|\Gamma_i\rangle$ and $|\Gamma_f\rangle$ are described by the model space wave functions. Greek symbols are used to denote quantum numbers in coordinate space and isospace, i.e $\Gamma_i \equiv J_i T_i$, $\Gamma_f \equiv J_f T_f$ and $\Lambda \equiv JT$.

The role of the core and the truncated space can be taken into consideration through a microscopic theory, which combines shell model wave functions and configurations with higher energy as first order perturbation to describe *EJ* excitations: these are called core polarization effects. The reduced matrix elements of the electron scattering operator $\hat{O}_\Lambda$ is expressed as a sum of the model space (MS) contribution and the core polarization (CP) contribution, as follows:

$$\left\langle \Gamma_f \,|||\, \tilde{\hat{T}}_\Lambda \,|||\, \Gamma_i \right\rangle = \left\langle \Gamma_f \,|||\, \hat{T}_\Lambda \,|||\, \Gamma_i \right\rangle_{\mathrm{MS}} + \left\langle \Gamma_f \,|||\, \Delta\hat{T}_\Lambda \,|||\, \Gamma_i \right\rangle_{\mathrm{CP}} \quad (9)$$

which can be written as:

$$\left\langle \Gamma_f \,|||\, \tilde{\hat{T}}_\Lambda \,|||\, \Gamma_i \right\rangle = \sum_{\alpha\beta} X^{\Lambda}_{\Gamma_f\Gamma_i}(\alpha,\beta)\left[\left\langle \alpha \,|||\, \hat{T}_\Lambda \,|||\, \beta \right\rangle + \left\langle \alpha \,|||\, \Delta\hat{T}_\Lambda \,|||\, \beta \right\rangle\right] \quad (10)$$

According to the first-order perturbation theory, the single particle core-polarization term is given by [10]:

$$\left\langle \alpha \,|||\, \Delta\widetilde{\hat{T}}_{\Lambda} \,|||\, \beta \right\rangle = \left\langle \alpha \,|||\, \hat{T}_{\Lambda} \frac{Q}{E_i - H_0} \mathrm{V}_{res} \,|||\, \beta \right\rangle + \left\langle \alpha \,|||\, \mathrm{V}_{res} \frac{Q}{E_f - H_0} \hat{T}_{\Lambda} \,|||\, \beta \right\rangle \quad (11)$$

where the operator Q is the projection operator onto the space outside the model space.
The single particle core-polarization terms given in equation (11) are written as [10]:

$$\left\langle \alpha \,|||\, \Delta\hat{T}_{\Lambda} \,|||\, \beta \right\rangle = \sum_{\alpha_1\alpha_2\Gamma} \frac{(-1)^{\beta+\alpha_2+\Gamma}}{\varepsilon_\beta - \varepsilon_\alpha - \varepsilon_{\alpha_1} + \varepsilon_{\alpha_2}} (2\Gamma+1) \begin{Bmatrix} \alpha & \beta & \Lambda \\ \alpha_2 & \alpha_1 & \Gamma \end{Bmatrix} \sqrt{(1+\delta_{\alpha_1\alpha})(1+\delta_{\alpha_2\beta})} \times \left\langle \alpha\alpha_1 \,|\, \mathrm{V}_{res} \,|\, \beta\alpha_2 \right\rangle_\Gamma \left\langle \alpha_2 \,|||\, \hat{T}_{\Lambda} \,|||\, \alpha_1 \right\rangle$$

+ terms with α_1 and α_2 exchanged with an overall minus sign (12)

where the index α_1 runs over particle states and α_2 over hole states and ε is the single-particle energy, and is calculated according to [10]:

$$\varepsilon_{n\ell j} = (2n+\ell-1/2)\hbar\omega + \begin{cases} -\frac{1}{2}(\ell+1)\langle f(r)\rangle_{n\ell} & \text{for } j=\ell-1/2 \\ \frac{1}{2}\ell\langle f(r)\rangle_{n\ell} & \text{for } j=\ell+1/2 \end{cases} \quad (13)$$

with $\langle f(r)\rangle_{n\ell} \approx -20A^{-2/3}$ and $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$

For the residual two-body interaction V_{res}, the Michigan sum of three range Yukawa potential (M3Y) interaction of Bertsch et al. [11] is adopted. The form of the potential is defined in equations (1)-(3) in Ref. [11]. The parameters of 'Elliot' are used which are given in Table 1 of the mentioned reference. A transformation between *LS* and *jj*is used to get the relation between the two-body shell model matrix elements and the relative and center of mass coordinates, using the harmonic oscillator radial wave functions with Talmi-Moshinsky transformation [12,13].

Using Wickner-Eckart theorem, the single particle matrix elements reduced in both spin and isospin, are written in terms of the single-particle matrix elements reduced in spin only:

$$\left\langle \alpha_2 \,|||\, \hat{T}_{\Lambda} \,|||\, \alpha_1 \right\rangle = \sqrt{\frac{2T+1}{2}} \sum_{t_z} I_T(t_z) \left\langle j_2 \,\|\, \hat{T}_{jt_z} \,\|\, j_1 \right\rangle \quad (14)$$

with: $$I_T(t_z) = \begin{cases} 1 & \text{for} \quad T=0 \\ (-1)^{1/2-t_z} & \text{for} \quad T=1 \end{cases} \quad (15)$$

where t_z= 1/2 for a proton and -1/2 for a neutron. The single particle matrix element of the electric transition operator reduced in spin is:

$$\left\langle j_2 \,\|\, \hat{T}_{Jt_z} \,\|\, j_1 \right\rangle = e_{t_z} \left\langle j_2 \| Y_J \| j_1 \right\rangle \left\langle n_2\ell_2 | j_J(qr) | n_1\ell_1 \right\rangle \quad (16)$$

where $|n\ell\rangle$ is the single-particle radial wave function.

The reduced single-particle matrix element of the longitudinal operator becomes:

$$\left\langle \alpha_2 \parallel\parallel \hat{T}_\Lambda \parallel\parallel \alpha_1 \right\rangle = e_T \sqrt{2(2T+1)} \left\langle j_2 \middle\| Y_J \middle\| j_1 \right\rangle \left\langle n_2 \ell_2 \middle| j_J(qr) \middle| n_1 \ell_1 \right\rangle \quad (17)$$

where e_T is the isoscalar ($T = 0$) and isovector ($T = 1$) charges.

Electron scattering form factor involving angular momentum J and momentum transfer q, between initial and final nuclear shell model states of spin $J_{i,f}$ and isospin $T_{i,f}$ are [14]:

$$|F_J(q)|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left| \sum_{T=0,1} (-1)^{T_i - T_z} \begin{pmatrix} T_f & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} \left\langle J_f T_f \parallel\parallel \tilde{\hat{T}}_{JT} \parallel\parallel J_i T_i \right\rangle \right|^2 F_{cm}^2(q) F_{fs}^2(q) \quad (18)$$

where $F_{cm}(q)$ is the center of mass correction which is given by $F_{cm}(q) = e^{-q^2 b^2/4A}$, with b is the harmonic oscillator size parameter and $F_{fs}(q)$ is the finite size correction given by $F_{fs}(q) = \left[1 + (q/4.33 fm^{-1})^2\right]^{-2}$ [15].

The reduced electromagnetic transition probability $B(CJ\uparrow)$ can be obtained from the longitudinal form factor evaluated at $q = k = \frac{E_x}{\hbar c}$ (photon point) as [16]:

$$B(CJ\uparrow) = \frac{Z^2}{4\pi} \left[\frac{(2J+1)!!}{k^J} \right]^2 |F_J(k)|^2 \quad (19)$$

The relation between the $B(CJ)$ values for the emission ↓ and absorption ↑ process is [10]:

$$B(CJ\downarrow) = \frac{2J_i+1}{2J_f+1} B(CJ\uparrow) \quad (20)$$

where i and f are the initial and final states, respectively.

For electromagnetic transition, the $B(EJ)$ value can be calculated directly in terms of the electric multipole transition operator [10]:

$$\hat{O}_{JM}(\vec{r}) = \sum_{k=1}^{n} e(k) r_k^J Y_{JM}(\Omega_k)$$

So, replacing the operator T in all the above equations by the operator O, equation (17) for the reduced single particle matrix element becomes:

$$\left\langle \alpha_2 \parallel\parallel \hat{O}_\Lambda \parallel\parallel \alpha_1 \right\rangle = e_T \sqrt{2(2T+1)} \left\langle j_2 \middle\| Y_J \middle\| j_1 \right\rangle \left\langle n_2 \ell_2 \middle| r^J \middle| n_1 \ell_1 \right\rangle \quad (21)$$

The reduced electromagnetic transition probability $B(EJ)$ is defined as [10]:

$$B(EJ) = \frac{1}{(2J_i+1)} \left| \sum_{T=0,1} (-1)^{T_i - T_z} \begin{pmatrix} T_f & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} \left\langle J_f T_f \parallel\parallel \tilde{\hat{O}}_{JT} \parallel\parallel J_i T_i \right\rangle \right|^2$$

which can be written as:

$$B(EJ) = \frac{1}{(2J_i+1)} \left| \sum_{T=0,1} e_T (-1)^{T_i - T_z} \begin{pmatrix} T_f & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} \left\langle J_f T_f \parallel\parallel \tilde{M}_{JT} \parallel\parallel J_i T_i \right\rangle \right|^2 \quad (22)$$

where $\tilde{M}_{JT} = \left\langle J_f T_f \parallel\parallel \tilde{\hat{M}}_{JT} \parallel\parallel J_i T_i \right\rangle$. The isoscalar ($T$=0) and isovector ($T$=1) charges are given by $e_0 = e_{IS} = \frac{1}{2}e$, $e_1 = e_{IV} = \frac{1}{2}e$.

The reduced electromagnetic transition probability can be represented in terms of only the model space matrix elements, but with effective charges, as:

$$B(EJ)=\frac{1}{(2J_i+1)}\left|\sum_{T=0,1} e_T^{eff}(-1)^{T_i-T_z}\begin{pmatrix} T_f & T & T_i \\ -T_z & 0 & T_z \end{pmatrix}\left\langle J_f T_f \parallel\mid \hat{M}_{JT} \parallel\mid J_i T_i \right\rangle\right|^2 \quad (23)$$

Then the isoscalar and isovector effective charges are given by:

$$e_T^{\text{eff}}=\frac{M_{JT}+\Delta M_{JT}}{2M_{JT}}e=\frac{e_p^{\text{eff}}+(-1)^T e_n^{\text{eff}}}{2} \quad (24)$$

The proton and neutron effective charges can be obtained as follows:

$e_{\text{p}}^{\text{eff}}=e_0^{\text{eff}}+e_1^{\text{eff}}$ and $e_n^{\text{eff}}=e_0^{\text{eff}}-e_1^{\text{eff}}$

The above effective charges work for mixed isoscalar and isovector transitions. For pure isoscalar transition, the polarization charge δe is given by:

$$\delta e=\frac{\Delta M_J}{2M_J}e \quad (25)$$

and the effective charges for the proton and neutron becomes

$$e_{\text{p}}^{\text{eff}}=e+\delta e, \quad e_{\text{n}}^{eff}=\delta e \quad (26)$$

The longitudinal form factor $F_J(q)$ can be written as:

$$F_J(q)=\frac{4\pi}{N_{t_Z}}=\int_0^\infty dr\, r^2\, j_J(qr)\,\rho_{J,t_Z}(r) \quad (27)$$

where the normalization factor N_{t_Z} is defined as:

$$N_{t_Z}=\left\{\begin{matrix} A\,(mass\,number) \\ Z\,(proton\,number) \\ N\,(neutron\ number) \end{matrix}\right\}$$

From equations (18) and (27), the nucleon transition density can be found to be [9]:

$$\rho_{J,t_Z}(r)=\frac{1}{4\pi}\sqrt{\frac{4\pi}{2J_i+1}}\sum_{a,b} OBDM(J_i,J_f,a,b,J,t_Z)$$
$$\times\ \left\langle n_a\frac{1}{2}j_a\left\|Y_J(\Omega_r)\right\|n_b\frac{1}{2}j_b\right\rangle R_{n_a}l_{n_a}(r)\,R_{n_b}l_{n_b}(r) \quad (28)$$

The corresponding mean square radius is given in terms of the nucleon density as [17]:

$$\left\langle r^2\right\rangle=\frac{4\pi}{A}\int_0^\infty \rho(r)\,r^4\,dr \quad (29)$$

or it is given in terms of the occupation number as :

$$\left\langle r^2\right\rangle=\frac{1}{A}\sum_{a,t_Z} n_a,t_Z(t_Z,j_a)(2n_a+l_a-\frac{1}{2})b^2 \quad (30)$$

where $n_a,t_Z(t_Z,j_a)$ is the average occupation number in each orbit.

Results and Discussion:

The single-particle wave functions of the harmonic oscillator (HO) potential are used with size parameter *b* that plays the role of a characteristic length of the harmonic-oscillator potential. This parameter is chosen to reproduce the measured ground state root mean square matter radii for all Ne-isotopes under consideration in this work by using relation (30). The reduced electromagnetic transition probabilities B(E2) from the first excited 2^+ state to the ground state are expressed in terms of the electron scattering form factors evaluated at *q*=*k*, where $k = E_x / \hbar c$, E_x is the excitation energy of the state. These B(E2) values represent basic nuclear information complementary to our knowledge of the energies of low-lying levels in these isotopes. The calculation of B(E2) for the isotopes considered in this work are performed by using the relation (19), where the calculation of the form factors are taken into account at $q \to 0$, and the relation between B(EJ↑) and B(EJ↓) is given by relation (20). The one-body density matrix element (OBDM) values are obtained by the shell model calculations that performed via the computer code OXBASH [18] using the SD shell model space with Universal SD-shell interaction B (USDB)[19].

The Nucleus ^{18}Ne:

^{18}Ne nucleus composed of the core ^{16}O nucleus plus two protons surrounding the core. These outer two protons are considered to move over the sd-shell model space. The ground state of this unstable nucleus is $J^{\pi}T = 0^{+}\,1$, with half-life $\tau = 1672\,ms$ [20]. The sd-shell model space with USDB interaction [19] is used to generate the OBDM elements for the ground state with *J*=0 and excited state with *J*=2. The harmonic oscillator size parameter is chosen to reproduce the rms matter radius. The size parameter is fixed at 1.818 fm , which is reproducing the measured value 2.81 ± 0.14 fm [5]. The ground state configurations show that two protons distributed outside the core over sd shell orbits with 77.62% over $1d_{5/2}$ orbit, 4.91% over $1d_{3/2}$ orbit and 17.47% over $2s_{1/2}$ orbit. The first excited 2^+ state of this nucleus is at 1.998 MeV. The calculations of the electric transition strength $B(E2;2_1^+ \to 0_1^+)$ by using the core polarization effects with the two body Michigan sum of three range Yukawa potential (M3Y) as a residual interaction [11] with excitation up to $2\hbar\omega$, gives the value of 25.078 e^2 fm^4. The effective nucleon charges that deduced and used are 1.154 e and 0.401 e for the proton and neutron, respectively. The calculations are performed by using the standard effective nucleon charges which are 1.3 e for the proton and 0.5 e for the neutron and the deduced transition strength value is 31.824 e^2 fm^4. There is an excellent agreement between the calculated value of the transition strength B(E2↓) of ^{18}Ne nucleus, in terms of the core-polarization effects, and the measured value 23 ± 4 e^2 fm^4 [21] than the value that calculated in terms of the standard nucleon charges, where this value is overestimates the measured value by about a factor of 1.3. The calculated and experimental values are tabulated in Table 1. The analysis of the above calculations shows a strong contribution of $1d_{5/2}$ orbit for valence two protons, and that is so apparent from Figure-1a.

The Nucleus ^{20}Ne:

This isotope composed of the core ^{16}O nucleus plus two protons and two neutrons distributed over the sd-shell model space. The ground state of stable ^{20}Ne isotope is $J^{\pi}T = 0^{+}\,1$. USDB interaction [19], is adopted to generate the OBDM elements for the multipolarities*J*=0 and *J*=2. The single-particle wave functions of a harmonic oscillator potential is used with size parameter *b*=1.815 fm, to reproduce the rms matter radius 2.87±0.03 fm [5]. Two sets of the nucleon effective charges are used to calculate the electric transition strengths $B(E2;2_1^+ \to 0_1^+)$ of ^{20}Ne nucleus from the first excited 2^+ state with excitation energy 1.811 MeV to the ground 0^+state. The first set is the effective nucleon charges, which are 1.3 e for the proton and 0.3 e for the neutron, deduced by adopting the core polarization calculation with M3Y interaction [11] and particle-hole excitation up to $2\hbar\omega$. The second set is the standard nucleon charges, which are 1.3 e and 0.5 e for the proton and neutron, respectively. It is so apparent from the Table 1 that the calculated B(E2↓) in terms of the standard nucleon charges is closer than that of the effective nucleon charges to explain the measured value 66 ± 3.8 e^2 fm^4[22]. There is a major contribution of $1d_{5/2}$ orbit for valence four particles, as that shown from Figure-1b, where the occupation number percentagesare calculated and presented for the ground states of $1d_{3/2}$, $1d_{5/2}$ and $2s_{1/2}$ orbits.

The Nucleus ^{22}Ne:

The electric transition strengths $B(E2;2_1^+ \rightarrow 0_1^+)$ of ^{22}Ne nucleus from the first excited 2^+ state with excitation energy 1.457 MeV to the ground 0^+ state are calculated with sd-shell model space, where the ground state of ^{22}Ne stable isotope is $J^\pi T = 0^+ 1$ and there are six nucleons considered to distribute over the sd-shell orbits outside ^{16}O core. USDB interaction [19] is used to generate the OBDM elements for the multipolarities$J$=0 and J=2. The single-particle wave functions of a harmonic oscillator potential is used with size parameter b=1.783 fm, to reproduce theoretical rms matter radius 2.87 fm [23]. The calculated value of the transition strength is 40.880 e^2 fm^4 which is obtained in terms of the core polarization effective charges 1.201 e for the proton and 0.442 e for the neutron that deduced by adopting the core polarization calculation with M3Y interaction [11] and particle-hole excitation up to $2\hbar\omega$. Another calculated value of the transition strength 49.304 e^2 fm^4 obtained by introducing a standard nucleon charges 1.3 e for the proton and 0.5 e for the neutron. As shown from Table 1 which contains all calculated and measured values, there is a reasonable agreement between the both calculated values of B(E2↓) and the measured value 46.4±0.6 e^2 fm^4[22]. There is a major contribution of $1d_{5/2}$ orbit for valence six particles outside ^{16}O core nucleus, as that shown from Figure-1c.

The Nucleus ^{24}Ne:

The single-particle wave functions of a harmonic oscillator potential is used with size parameter 1.708fm to reproduce the rms matter radius 2.79±0.13 fm [5], where the ground state of this unstable isotope is $J^\pi T = 0^+ 2$, with a half-life of $\tau = 3.38\,m$ [20]. This unstable nucleus composed of the core ^{16}O nucleus plus eight particles (two protons and six neutrons) surrounding the core. These outer eight particles are considered to move over the sd-shell model space. USDB interaction [19], is adopted to generate the OBDM elements for both multipolarities$J$=0 and J=2. Core polarization calculation, with M3Y interaction [11] and particle-hole excitation up to $2\hbar\omega$, gives the value 28.285 e^2 fm^4 of the electric transition strength $B(E2;2_1^+ \rightarrow 0_1^+)$ of ^{24}Ne nucleus from the first excited 2^+ state with excitation energy 1.981 MeV to the ground 0^+ state, where the deduced effective nucleon charges that applied are 1.292 e and 0.409 e for the proton and neutron, respectively. The standard effective nucleon charges that applied are 1.3 e for the proton and 0.5 e for the neutron, gives the value 32.122 e^2 fm^4 of B(E2↓). All calculated and measured values are tabulated in Table 1, and it so clear that the calculated B(E2↓) in terms of the core-polarization effects matches the measured value 28.2±6.4 e^2 fm^4 [22], and the calculated value that deduced in terms of the standard nucleon charges is a good value and locating within the range of the measured value. A strong contribution of $1d_{5/2}$ orbit for outer eight particles is so clear from Figure -1d.

The Nucleus ^{26}Ne:

The size parameter *b* of the used single-particle wave function of the harmonic oscillator potential is fixed to be 1.730 fm to reproduce the measured rms of the unstable ^{26}Ne isotope that equal to 2.86±0.05 fm [5]. The ground state of this isotope is $J^{\pi}T = 0^{+}\,3$, with a half-life of $\tau = 192\,ms$ [20]. ^{26}Ne nucleus composed of ^{16}O nucleus that considered as a core, plus ten nucleons (two protons and eight neutrons) distributed over the sd-shell model space. The OBDM elements for both multipolarities*J*=0 and *J*=2 are calculated in terms of the USDB interaction [19]. Two values of the electric transition strength $B(E2;2_1^+ \rightarrow 0_1^+)$ of ^{26}Ne nucleus from the first excited 2^+ state with excitation energy 2.018 MeV to the ground 0^+ state, are calculated and compared with the measured value 45.6±8.2 e^2 fm^4 [22], as well as with each other as that shown in Table 1. The first value 27.116 e^2 fm^4 is calculated according to introducing the core polarization calculation, with M3Y interaction [11] and particle-hole excitation up to $2\hbar\omega$, where the effective nucleon charges that obtained and used to calculate the B(E2↓) are 1.167 e and 0.505 e for the proton and neutron, respectively. The second value 32.059 e^2 fm^4 deduced with using the standard effective nucleon charges which are 1.3 e for the proton and 0.5 e for the neutron. Both calculated values are underestimating the measured value by factors of 1.6 and 1.4 for the calculation in terms of the core-polarization effective charges and standard effective charges, respectively. Finally, and for the purpose of explaining the measured value, we can say that the calculated B(E2↓) value in terms of the standard charges is closer than the calculated value obtained in terms of the core-polarization effects. Figure -1e shows a strong contribution of $1d_{5/2}$ orbit for the outer ten particles.

The Nucleus ^{28}Ne:

The ground state of this unstable isotope is $J^{\pi}T = 0^{+}\,4$, with a half-life of $\tau = 19\,m$ [20]. This nucleus composed of the core ^{16}O nucleus plus twelve particles (two protons and ten neutrons) distributed over the sd-shell model space. The single-particle wave functions of a harmonic oscillator potential is used with size parameter *b*=1.749fm that fixed to reproduce the rms matter radius 2.92±0.10 fm [5]. The OBDM elements are calculated via the computer code OXBASH [18] using the USDB interaction [19], for both multipolarities*J*=0 and *J*=2. The electric transition strengths B(E2↓) of ^{28}Ne nucleus from the first excited 2^+ state ($J_f^{\pi}\,T_f = 2^{+}\,4$) with excitation energy 1.879 MeV to the ground 0^+ state ($J_i^{\pi}\,T_i = 0^{+}\,4$), are calculated in terms of the core-polarization effective nucleon charges 1.165 e for the proton and 0.505 e for the neutron, and in terms of the standard nucleon charges 1.3 e for the proton and 0.5 e for the neutron. The core-polarization effective charges are deduced by adopting the two body Michigan sum of three range Yukawa potential (M3Y) [11] as a residual two body interaction with particle-hole excitation up to $2\hbar\omega$, where the obtained values that mentioned above are introduced in the calculations and gives B(E2↓) equal to 24.206 e^2 fm^4. Using the standard nucleon charges gives B(E2↓) equal to 28.509 e^2 fm^4. There is a good agreement between both calculated values of the reduced transition strength and the measured value 53.8±27.2 e^2 fm^4[22,24], as shown from Table 1 that containing all calculated and experimental values. There is a major contribution of $1d_{5/2}$ orbit for valence twelve particles, as in Figure-1f, where the occupationpercents are calculated and presented for the ground states of $1d_{3/2}$, $1d_{5/2}$ and $2s_{1/2}$ orbits.

Table 1 - The calculated and experimental electric transition strengths and effective charges of the considered Ne – isotopes.

A τ	**Ex. (MeV)** **Calc.**	**Exp.[25]**	***b* (fm)**	**Eff. Charges** e_p **(e)**	e_n **(e)**	**B(E2↓) e^2 fm^4** **Calc.**	**Exp.**
18 1672 ms	1.998	1.887	1.818	1.154 1.3	0.401 0.5	25.078 31.824	23 ± 4 [21]
20 Stable	1.811	1.633	1.815	1.3 1.3	0.3 0.5	42.571 53.878	66.8±3.8 [22]
22 Stable	1.457	1.274	1.783	1.201 1.3	0.442 0.5	40.880 49.304	46.4±0.6 [22]
24 3.38 m	1.981	2.292	1.708	1.292 1.3	0.409 0.5	28.285 32.122	28.2±6.4 [22]
26 192ms	2.018	2.227	1.730	1.167 1.3	0.505 0.5	27.116 32.059	45.6±8.2 [22]
28 19ms	1.879	1.310	1.749	1.165 1.3	0.505 0.5	24.206 28.509	53.8±27.2 [22,24]

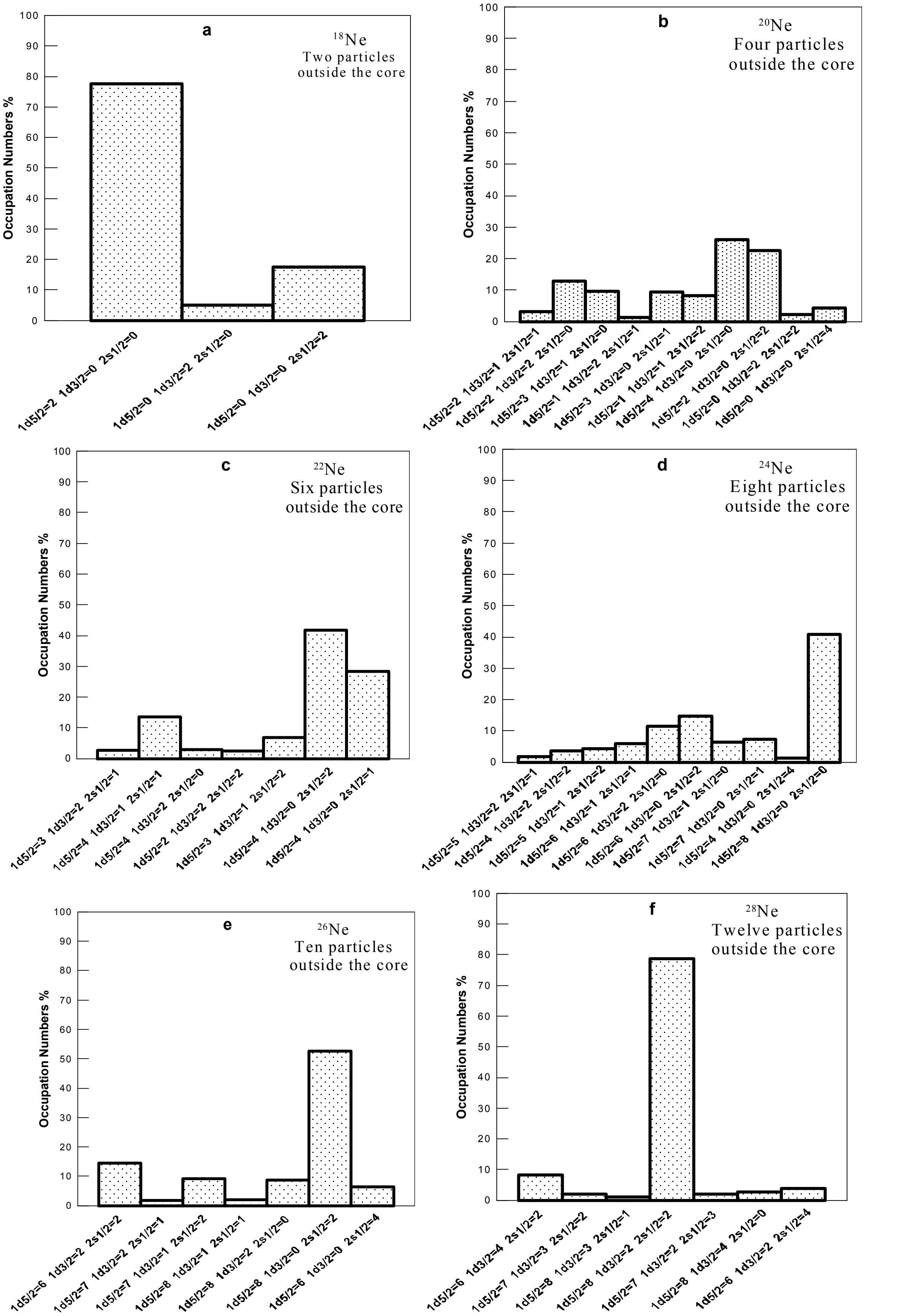

Figure 1- The percent occupation for the ground states of $1d_{3/2}$, $1d_{5/2}$ and $2s_{1/2}$ orbits outside ^{16}O core of the considered Ne – isotopes.

Conclusions:

Shell model calculations are performed over some even-even Neon isotopes. The size parameter *b* of the HO potential is fixed to reproduce the measured rms matter radii, and used to calculate the transition strengths B(E2↓), where the transition strengths depends strongly on *b*.

The quadrupole excitation is interested in this work, and the transition strengths $B(E2;2_1^+ \rightarrow 0_1^+)$ of the considered nuclei from the first 2^+ excited states to the ground 0^+ states are calculated.

The core contributions cannot be ignored in such transitions and the core polarization effects play a major role for such transitions. The inclusion of core polarization effects with excitation up to $2\hbar\omega$, gives a remarkable improvement in the transition strengths, and describes the data remarkably well.

Two sets of nucleon charges are adopted in the calculation of the transition strengths differ from the nucleon bare charges $e_p = 1.0\,e$ and $e_n = 0.0\,e$ which gives results underestimate the measured values. The first set is obtained in terms of the core polarization effects, and the second set is the standard nucleon charges which are 1.3 e for the proton and 0.5 e for the neutron. A good agreement between the calculated values of B(E2↓) in terms of the core polarization effects, and the experimental values is obtained for most considered isotopes, but for ^{20,26}Ne isotopes there is a slight underestimation of the measured values, where the calculated values in terms of the standard nucleon charges are closer to theinterpretation of the measured results.

The analysis of the calculations for the considered isotopes shows a strong contribution of $1d_{5/2}$ orbit and a clear exotic behavior for the valence nucleons outside ^{16}O core.

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