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Centralizer and Jordan Centralizer of Inverse Semirings

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Abstract
Let M be a semiprime 2-torsion free inverse semiring, and let α be an endomorphism of M. Under some conditions, we prove a Jordan α-centralizer of M is a α-centralizer of M, also we prove if R: $M \to M$ be an additive mapping such that $R(r^3) + \alpha(r)R(r)\alpha(r)' = 0$ holds for all $r \in M$, where R is a centralizer, and α is a surjective endomorphism of M.

Keywords: Inverse semiring, centralizer of inverse semiring, Jordan centralizer, α-centralizer, Jordan α-centralizer.

تمركزات و تمركزات جوردان لاشباه الحلقات المعكوسة

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الخلاصة
في ضل بعض الظروف، نثبت ان Mهو تشاكل على α شبه حلقه معكوسه شبه اوليه2 طليقه الالتواء، و Mلتكن تمركزات−α جوردان ل M تكون تمركزات جوردان ل M , نبرهن ايظا اذا R: $M \to M$ تكون تطبيق جمعي بحيث
$R(r^3) + \alpha(r)R(r)\alpha(r)' = 0$ لكل $r \in M$. في هذه الحالة تكون R تمركز، عندما α تكون تشاكل شامل على M.

1. Introduction

The investigation of the semiring goes back to Vandiver [1]. A non-empty set with two binary operation (+) and (•) is called semiring if and only if the following conditions hold:
i) $(M, +)$ is commutative semigroup.
ii) $(M, \bullet)$ is semigroup.
iii) $a \bullet (r + s) = a \bullet r + a \bullet s$, and $(r + s) \bullet a = r \bullet a + s \bullet a$
for all $a, r, s \in M$.

A semiring $(M, +, \bullet)$ is said to be commutative if and only if $r \bullet s = s \bullet r$ holds for all $r, s \in M$, and it's called additively inverse semiring, if for every $r \in M$ there exists a unique element $r' \in M$ such that $r + r' + r = r$ and $r' + r + r' = r'$, [2]. The semiring M is known as a semiring with 0, if there exists an element $0 \in M$ such that $r + 0 = r$ for all $r \in M$, and is known as a semiring with unity, if there exists an element $1 \in M$ such that $r \bullet 1 = 1 \bullet r = r$ for

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all $r \in M$, [3]. A semiring M is additively left cancellative if for all $r, s, m \in M$, such that $r + s = r + m$, then $s = m$, and is additively right cancellative if $s + r = m + r$, then $s = m$, [3].

In this article, M will represent additive inverse semiring that satisfies the condition, for all $r \in M$, $r + \acute{r}$ is located in the center Z (M) of M.

A semiring M is called a prime semiring if for any $r, s \in M$, if $rMs = 0$ implies either $r = 0$ or $s = 0$, M is semiprime if $r M r = 0$, implies that $r = 0$, and M is m-torsion free if $mr = 0$, $r \in M$ implies $r = 0$. A commutator [., .] in an inverse semirings defines as [r, s] = $rs + rs'$ and, r o $s = rs + rs$, [4].

An inverse semiring M is said to be has a commutator which is not left (right) zero divisor if there exists $r, s \in M$ such that $[r, s]\, t = 0$, $(t\, [r, s] = 0)$, $t \in M$, implies that $t = 0$, [5]. We call a map $d: M \to M$ a derivation, when $d(rs) = d(r)s + rd(s)$ holds for all $r, s \in M$, and we call it a Jordan derivation when $d(r^2) = d(r)r + rd(r)$ holds for all $r \in M$. An additive mapping $R : M \to M$ is called a left (right) centralizer in case $R(rs) = R(r)s$ $(R(rs) = r\, R(s))$ holds for all $r, s \in M$. We follow Zalar [6] and refer to R as a centralizer when it is both a left and a right centralizer. An additive mapping R: $M \to M$ is called a left (right) Jordan centralizer in case $R(r^2) = R(r)r$, $(R(r^2) = rR(r))$.

In [7] Albas introduced the α-centralizer notation and the Jordan α-centralizer notation, which are a generalization of Jordan centralizer and centralizer, and tested under specific conditions on a 2-torsion free semiprime ring, where every Jordan α-centralizer is α-centralizer, and where α is a surjective homomorphism. An inverse semirings considered in different directions by numerous authors (see for example [8-16]). In this work our aim is to consider the results of Majeed and Meften [17, 18] in the inverse semiring.

2. α-Centralizer of inverse semiring

In this section we present the definition of left (right) α-centralizer, left (right) Jordan α-centralizer of a semiring M, and some lemmas that will be used later.

Definition 2.1

A left (right) α-centralizer of a semiring M is an additive mapping R: $M \to M$ which satisfies $R(rs) + R(r)\alpha(s)' = 0$, $(R(rs) + \alpha(r)R(s)' = 0)$ for all $r, s \in M$, a α-centralizer of a ring M is both left and right α-entralizer, where α is an additive mapping on M.

Definition 2.2

A left (right) Jordan α-centralizer of a semiring M is an additive mapping R: $M \to M$ which satisfies $R(r^2) + R(r)\alpha(r)' = 0$ $(R(r^2) + \alpha(r)\, R(r)' = 0)$ for all $r \in M$. A Jordan α-centralizer of a ring M is both left and right Jordan α-centralizer, where α is an additive mapping on M.

Clearly, the left α-centralizer of M is the left Jordan α-centralizer and similarly, the α -centralizer for M is the Jordan α-centralizer for M.

Lemma 2.3 [10, 11]

Let M be an inverse semiring, then for all $r, s \in M, r + s = 0,$ and $r = s'$.
Note that in general $r + r' \neq 0, r + r' = 0$, if and only if there exists some $s \in M$ with $r + s = 0$.

Lemma 2.4 [10]
For all $r, s \epsilon M$, the following are holds:
i. $(r + s)' = r' + s'$.
ii. $(rs)' = r's = rs'$.
iii. $r'' = r$.
iv. $r's' = (r's)' = (rs)'' = rs$.

Lemma 2.5 [12]
For all $r, \ s, t \epsilon M$, the following are holds:
i. $[r , r] = 0$.
ii. $[r + s , t] = [r , t] + [r , s]$.
iii. $[rs , t] = r[s , t] + [r , t]s$.
iv. $[r , st] = s[r , t] + [r , s]t$.

Lemma 2.6 [10]
Let M be a semiprime, if $r, s \epsilon M$ such that $rxs = 0$ for all $x \in M$. Then $rs = sr = 0$.

Recall that an additive map is an additive in each argument.

Lemma 2.7 [17]
Let M be a semiprime and F, G: $M \times M \rightarrow M$ be additive mappings. If $F(r,s)wG(r,s) = 0\ for\ all\ r,s,w\ \epsilon M,$ then $F(r,s)wG(t,q) = 0\ for\ all\ r,s,w,t,q\ \epsilon M.$

3. **Main Results**
The following theorem is the generalization of the theorem in [17].

Theorem 3.1
Let M be 2-torsion free semiring, following each left (right) Jordan $\alpha -$ centralizer. If one of the following statements about R is true, then R is a left (right) $\alpha -$ centralizer:

i M is a semiprime semiring has a commutator which is not a zero divisor.
ii M is a non commutative prime semiring.
iii M is a commutative semiprime semiring.
Where α is a surjective endomorphism of M.
Proof :

$$R(r^2) + R(r)\alpha(r)' = 0 \qquad for\ all\ r \in M \tag{1}$$

If we replace $r\ by\ r + s$, we get

$$R((r+s)^2) + R(r+s)\alpha(r+s)' = 0$$

By Definition 2.1, we have some terms became = 0

$$R(r^2) + R(rs+sr) + R(s^2) + R(r)\alpha(r)' + R(r)\alpha(s)' + R(s)\alpha(r)' + R(s)\alpha(s)' = 0$$

$$R(rs+sr) + R(r)\alpha(s)' + R(s)\alpha(r)' = 0 \tag{2}$$

$$R(rs) + R(r)\alpha(s)' + R(sr) + R(s)\alpha(r)' = 0$$

$$F(r,s) + F(s,r) = 0$$

$$F(r,s) = F(s,r)'.$$

Where $F(r,s) = R(rs) + R(r)\alpha'(s)$ and $F(s,r) = R(sr) + R(s)\alpha(r)'\ for\ all\ r,s \in M.$
By replacing s with $rs + sr$ and using (2), we arrive at

$$R(r(rs+sr) + (rs+sr)r) + R(r)\alpha(rs+sr)' + R(rs+sr)\alpha(r)' = 0$$

$$R(r(rs+sr) + (rs+sr)r) + R(r)\alpha(rs)' + R(r)(sr)' + R(r)\alpha(sr)' + R(s)\alpha(r^2)' = 0 \tag{3}$$

In other way using equation (2), we have

$$R(r(rs + sr) + (rs + sr)r) + R(r)\alpha(rs)' + R(s)\alpha(r^2)' + 2R(rsr)' = 0 \quad (4)$$

Comparing (3) and (4), we obtain

$$R(rsr) + R(r)'\alpha(sr) = 0 \qquad for\ all\ r, s \in M \quad (5)$$

If we linearize (5), we get

$$R(rsz + zsr) + R(r)'\alpha(sz) + R(z)'\alpha(sr) = 0 \qquad for\ all\ r, s \in M \quad (6)$$

Now we shall compute $K = R(rszsr + srzrs)$ for all $r, s, z \in M$ in two different ways. Using (5) we have

$$K\ R(r)\alpha(szsr) + R(s)\alpha(rzrs) \quad (7)$$

using (6), we have

$$K\ R(rs)\alpha(zsr) + R(sr)\alpha(zrs) \quad (8)$$

comparing (7) and (8)

$$R(r)\alpha(szsr) + R(s)\alpha(rzrs) = R(rs)\,\alpha(zsr) + R(sr)\,\alpha(zrs)$$

$$R(r)\alpha(s)'\alpha(zsr) + R(rs)\alpha\,(zsr) + R(s)\alpha(r)'\alpha(zrs) + R(sr)\,\alpha(zrs) = 0$$

since $F(r, s)$ is a additive mapping, we arrive at

$$F(r, s)\alpha(zsr) + F(s, r)\alpha(zrs) = 0\ or\ all\ r, s, z \in M \quad (9)$$

Since $F(r, s) = F(s, r)'$ for all $r, s \in M$, using this fact and equality (9), we obtain

$$F(r, s)\alpha(z)[\alpha(r), \alpha(s)] = 0 \qquad for\ all\ r, s, z \in M \quad (10)$$

Using Lemma (2.7), we have

$$F(r, s)\alpha(z)[\alpha(u), \alpha(v)] = 0 \qquad for\ all\ r, s, z, u, v \in M \quad (11)$$

Using Lemma (2.6), we have

$$F(r, s)\,[\alpha(u), \alpha(v)] = 0 \qquad for\ all\ r, s, u, v \in M \quad (12)$$

(i) Where M has a commutator which is not a zero divisor using (12) and α is onto, we have

$$F(r, s) = 0 \quad \text{for all } r, s \in M$$

(ii) If M is a non commutative prime semiring using (11) and α is onto, we have

$$F(r, s) = 0 \qquad for\ all\ r, s \in M$$

(iii) If M is a commutative semiprime semiring, now we shall compute $J = R(rszsr)$ in two different ways, using (5)
we have

$$J = R(r)\alpha(szsr) \quad (13)$$

$$J = R(rs)\alpha(zsr) \quad (14)$$

Comparing (13) and (14), we arrive at

$$R(r)\alpha(szsr)' + R(rs)\alpha(zsr) = 0$$

$$(R(rs) + R(r)\alpha'(s))\,\alpha(z)\alpha(sr) = 0$$

$$F(r, s)\alpha(z)\alpha(sr) = 0 \qquad for\ all\ r, s, z \in M \quad (15)$$

Let $\psi(r, s) = \alpha(r)\alpha(s)$, it's clear that ψ is a additive mapping, therefore

$$F(r, s)\,\alpha(z)\,\psi(r, s) = 0 \qquad for\ all\ r, s, z \in M$$

Using Lemma (2.7), we have

$$F(r, s)\,\alpha(z)\,\psi(u, v) = 0 \qquad for\ all\ r, s, z, u, v \in M$$

Implies that

$$F(r, s)\,\alpha(z)\,\alpha(uv) = 0 \qquad for\ all\ r, s, z, u, v \in M \quad (16)$$

By replacing $\alpha(v)$ with $F(r, s)\alpha(z)$, α is onto, and M is a semiprime semiring, we have $F(r, s) = 0$ for all $r, s \in M$
That is mean

$$R(rs) + R(r)\alpha'(s) = 0.$$

By similar way we can prove

$$R(rs) + \alpha'(s)R(r) = 0.$$

Corollary 3.2

Let M be a 2-torsion free prime semiring, then every left (right) Jordan α-centralizer R is a left (right) α-centralizer.

In the following, we generalized the second proposition in [18], as following to α-centralizers for semiprime semiring, so to prove this result, we need the following lemmas

Lemma 3.3

Let M be a semiprime, $f\ is\ a\ (\alpha,\alpha)-derivation\ of\ M,$ and $a\in M$ some fixed element, where α is a surjective endomorphism of M

i. If $f(r)f(s)=0\ \ for\ all\ r,s\in M$ then $f=0$.

ii. If $ar + ra' \in Z(M)$ $for\ all\ r,s\in M$ then $a\in Z$.

Proof:

i. $f(r)\alpha(s)f(s)=f(r)f(sr)+f(r)f(s)\alpha(r)'=0 \qquad for\ all\ r,s\in M$

Sinceα is surjective, and M is a semiprime, we have $f = 0$

ii. Let $f(r)=a\alpha(r)+\alpha(r)a'd(r)$

It is clear that f is a (α, α) derivations, since $f(r)\in Z(M)$ for all $r\in M$, we get $f(s)\alpha(r) = \alpha(r)f(s)$ and also $f(sz)\alpha(r) = \alpha(r)f(sz)$.

Hence

$$f(s)\alpha(zr) + \alpha(s)f(z)\alpha(r) = f(s)\alpha(r)\alpha(z) + f(z)\alpha(r)\alpha(s)$$
$$f(s)\alpha(z)\alpha(r) + f(z)\alpha(s)\alpha(r) = f(s)\alpha(r)\alpha(z) + f(z)\alpha(r)\alpha(s)$$

Since M is semiring, we get

$$f(s)[\alpha(z), \alpha(r)]= f(z)[\alpha(r), \alpha(s)]$$

Since α is surjective take $\alpha(z) = a$. It is clear that $f(a) = 0$, so we obtain

$$f(s)[a, \alpha (r)] = f(s)f(a)=0$$

by (i) we get $f = 0$ and hence $a\alpha(r) + \alpha(r)a' =0$, for all $r\in M$.

Since α is a surjective, therefore $a \in Z(M)$.

Lemma 3.4

Let M be a semiprime semiring and $r\in M$ some fixed element. If $R(x)\ =\ r\alpha(x)\ +\ \alpha(x)$, and $R(x\ o\ s)\ =\ R(x)\ o\ \alpha\ (s)$ for all $x\ ,s\ \in\ M$. Then $r\ \in\ Z(M)$, where α is a surjective endomorphism of M.

Proof:

By the assumption

$$R(xs\ +\ sx)\ +\ R(x)\alpha(s)'\ +\ \alpha(s)'R(x)=0 \qquad for\ all\ x\ ,s\ \in\ M$$

Its clear that R is an additive map

$$R(xs)+R(sx)\ +\ R(x)\alpha(s)'\ +\ \alpha(s)'R(x)=0 \qquad for\ all\ x\ ,s\ \in\ M$$

On the other hand, we have

$$R(xs)+R(sx)\ =\ r\alpha(xs)+\alpha(xs)r+r\alpha(sx)+\alpha(sx)r$$
$$=r\alpha(x)\alpha(s)+\alpha(x)\alpha(s)r+r\alpha(s)\alpha(x)+\alpha(s)\alpha(x)r$$

$$R(x)\alpha(s)'+\alpha(s)R(x)'+r\alpha(x)\alpha(s)+\alpha(x)r\alpha(s)+\alpha(s)r\alpha(x)+\alpha(s)\alpha(x)r=0$$

$$(r+r')\alpha(x)\alpha(s)+\alpha(s)\alpha(x)(r+r')+\alpha(x)\alpha(s)+r\alpha(s)\alpha(x)+\alpha(x)r'\alpha(s)+\alpha(s)\,r'\,\alpha(x)\ =\ 0$$

Since $r\ +\ r'\ \in\ Z(R)$

$$\alpha(x)(r'+r+r')\alpha\ (s)+\alpha(s)(r'+r+r')\alpha\ (x)+\alpha\ (x)\alpha\ (s)r+r\alpha(s)\alpha(x)=0.$$

$$\alpha(x)r'\alpha\ (s)+\alpha(s)r'\alpha(x)+\alpha(x)\alpha(s)r+r\alpha(s)\alpha(x)=\ 0$$

$$\alpha(x)'\ (r\ \alpha(s)\ +\ \alpha(s)\ r')\ +(\ \alpha\ (s)r'+r\ \alpha\ (s))\ \alpha(x)=\ 0$$

Since α is surjective

$$r\alpha(s)+\alpha(s)r'\ \in\ Z(M)$$

The second part of Lemma (3.3) now gives we us $r \in Z(M)$.

Lemma 3.5

Let M be a semiprime semiring, $R: M \to M$ an additive map, which satisfies $R(xos) = R(x)o\alpha(s)$ $for\ all\ x, s \in M$, then $R(r) \in Z(M)$ for all $r \in Z(M)$, where α is a surjective endomorphism of *M*.

Proof:

Take $r \in Z(M)$ and denote $a = R(r)$.

$$2R(rx) = R(rx + xr) = R(r)\alpha(x) + \alpha(x)R(r) = a\alpha(x) + \alpha(x)a$$

Let

$$S(x) = 2R(cx)$$

is satisfies

$$S(xos) = 2R(r(xs + sx)$$
$$= 2R(rxs + srx)$$
$$= 2R(rx)\alpha(s) + 2\alpha(s)R(rx)$$
$$= S(x)\alpha(s) + \alpha(s)S(x)$$
$$= S(x)\ o\ \alpha\ (s)$$

$$S(xos) = 2R(r\ (xs + sx)$$
$$= 2R(x(rs) + (rs)x)$$
$$= 2\alpha(x)R((rs) + 2R(rs)\alpha(x)$$
$$= \alpha(x)S(s) + S(s)\alpha(x)$$
$$= \alpha(x)oS(s).$$

Therfore

$$S(xos) = S(x)o\alpha(s) = \alpha(x)oS(s) \quad for\ all\ x, s \in M$$

By Lemma (3.4), we have $a = R(r) \in Z(M)$.

Theorem 3.6

Let M be 2-torsion free and $R: M \to M$ be an additive map, which satisfies $R(xos) = R(x)o\alpha(s) = \alpha(x)oR(s)$ for all $x, s \in M$, then R is α-centralizer of *M*, if one of the following statements hold:

(i) M is a semiprime semiring has a commutator which is not zero divisor.

(ii) M is a non-commutative prime semiring.

(ii) M is a commutative semiprime semiring.

Where α is a surjective endomorphism of *M*, and α(*Z*(*R*)) = *Z*(*R*).

Proof:

$$R(xs + sx) = R(x)\alpha(s) + \alpha(s)R(x).$$
$$R(xs + sx) = \alpha(x)R(s) + R(s)\alpha(x)\ for\ all\ x, s \in Z(M).$$

If we replace *s* by *xs* + *sx*, we get

$$R(x)\alpha(xs + sx) + \alpha(xs + sx\ )R(x)$$
$$= \alpha(x)\ R(xs + sx) + R(xs + sx)\alpha(x)$$
$$= \alpha(x)R(x)\alpha(s) + \alpha(x)\alpha(s)R(x) + R(x)\alpha(s)\alpha(x) + \alpha(s)R(x)\alpha(x)$$
$$= R(x)\alpha(x)\alpha(s) + R(x)\alpha(s)\alpha(x) + \alpha(x)\alpha(s)R(x) + \alpha(s)\alpha(x)R(x).$$

Comparing the two above equations, we get

$$\big(\alpha'(x)R(x) + R(x)\alpha(x)\big)\alpha(s) = \alpha(s)\big(R(x)\alpha'(x) + \alpha(x)R(x)\big)\ for\ all\ x, s \in M.$$

Now it follows that

$$[R(x), \alpha(x)]\alpha(s) = \alpha(s)[R(x), \alpha(x)]\ \ holds \quad for\ all\ x, s \in M.$$

But α is surjective, then we get

$$[R(x), \alpha(x)] \in Z(M).$$

The next goal is to show that $[R(x), \alpha(x)] = 0$ holds *for all x ∈M.*

Take any $r \in Z(M)$.

$$2R(rx) = R(rx + xr)$$
$$= R(r)\alpha(x) + \alpha(x)R(r)$$
$$= 2R(r)\alpha(x).$$
$$2R(rx) = R(xr + rx)$$
$$= R(x)\alpha(r) + \alpha(r)R(x)$$
$$= 2R(x)\,\alpha(r).$$

Using Lemma 3.5, we get

$$R(rx) = R(x)\alpha(r) = R(r)\alpha(x) \qquad for\ all\ x \in M.$$
$$R(x)\alpha(r) + R(r)\,'\alpha(x) = 0$$
$$[R(x), \alpha\,(x)]\alpha(r) = R(x)\alpha(x)\,\alpha(r) + \alpha(x)R(x)'\,\alpha(r)$$
$$= R(x)\alpha(r)\,\alpha(x) + \alpha(x)R(r)'\alpha(x)$$
$$= (R(x)\alpha(r) + R(r)'\,\alpha(x))\,\alpha(x) = 0.$$

Since M is semiprime semiring, $\alpha\,(Z(M)) = Z(M)$, and $[R(x), \alpha(x)]$ itself is central element, we get

$$[R(x), \alpha(x)] = 0\ holds \qquad for\ all\ x \in M.$$
$$2R(x^2) = R(xx + xx) = R(x)\alpha(x) + \alpha(x)R(x)$$
$$= 2R(x)\alpha(x)$$
$$= 2\alpha(x)R(x) \qquad for\ all\ x \in M.$$

By Theorem 3.1, we get our result.

Corollary 3.7

Let M be 2-torsion free prime semiring and R: $M \to M$ an additive mapping which satisfies $R(x \circ s) = R(x) \circ \alpha(s) = \alpha(x) \circ R(s)$ for all $x, s \in M$, then R is a α-centralizer of M, where α is a surjective endomorphism of M, and α (Z (M)) = Z (M).

If M is prime ring, we get the following corollary:

Corollary 3.8

Let M be 2-torsion free prime semiring, then every left (right) Jordan α-centralizer is a left (right) α-centralizer, where α is a surjective endomorphism of M.

Theorem 3.9

Let M be a 2-torsion free semiprime with identity, and let R: $M \to M$ be an additive mapping. Suppose that $R(r^3) + \alpha(r)R(r)\alpha(r)' = 0$ holds for all $r \in M$. In this case, R is a $\alpha -$ centralizer, where α is a surjective endomorphism of M.

Proof:

Replacing r by $r + 1$ in the relation, where 1 is the identity element, we obtains

$$R((r+1)^3) + \alpha(r+1)R(r+1)\alpha(r+1)' = 0$$
$$R(r^3) + 3R(r^2) + 3R(r) + R(1) + \alpha(r)R(r)\alpha(r)' + \alpha(r)R(r)' + \alpha(r)R(1)\alpha(r)' + \alpha(r)R(1)' + R(r)\alpha(r)' + R(r)' + R(1)\alpha(r)' + R(1)' = 0$$

Using the assumption, we get

$$3R(r^2) + 2R(r) + R(1) + \alpha(r)R(r)' + \alpha(r)R(1)\alpha(r)' + \alpha(r)R(1)'$$
$$R(r)\alpha(r)' + R(1)\alpha(r)' + R(1)' = 0.$$

Putting r' for r in the relation above

$$3R(r^2) + 2R(r)' + R(1)\alpha(r)R(r)' + \alpha(r)R(1)\alpha(r) + \alpha(r)R(1) + R(r)\alpha(r)' + R(1)\alpha(r) + R(1)' = 0.$$

Comparing the two above relations, we obtain

$$6R(r^2) + 2\alpha(r)R(r)' + 2\alpha(r)R(1)\alpha(r)' + 2R(r)\alpha(r)' = 0 \qquad for\ all\ r \in M, \tag{17}$$

and

$$2R(r) + \alpha(r)R(1)' + R(1)\alpha(r)' = 0 \qquad for\ all\ r \in M. \qquad (18)$$

We want to prove that $R(1) \in Z(M)$. According to Eq. (18) one can replace $2R(r)$ on the right side of Eq. (17) by $\alpha(r)R(1) + R(1)\alpha(r)$ and $6R(r^2)$ on the left side by $3R(1)\alpha(r^2) + 3\alpha(r^2)R(1)$ which gives after some calculation

$$6R(r^2) = \alpha(r)(\alpha(r)R(1) + R(1)\alpha(r)) + 2\alpha(r)R(1)\alpha(r) + (\alpha(r)R(1) + R(1)\alpha(r))\alpha(r)$$

$$= \alpha(r^2)R(1) + \alpha(r)R(1)\alpha(r) + 2\alpha(r)R(1)\alpha(r) + \alpha(r)R(1)\alpha(r) + R(1)\alpha(r^2)$$

$$= \alpha(r^2)R(1) + 4\alpha(r)R(1)\alpha(r) + R(1)\alpha(r^2).$$

On the other hand from Eq. (17) and Eq. (18), we obtain

$$6R(r^2) = 3\alpha(r^2)R(1) + 3R(1)\alpha(r^2)$$

$$3\alpha(r^2)R(1) + \alpha(r^2)R(1)' + 3R(1)\alpha(r^2) + R(1)'\alpha(r^2) + 4\alpha(r)R(1)\alpha(r)' = 0.$$

After some calculation using the property of semiring, we get

$$\alpha(r^2)R(1) + R(1)\alpha(r^2) + 2\alpha(r)R(1)\alpha(r)' = 0 \qquad for\ all\ r \in M.$$

The above relation can be written in the for

$$[[R(1), \alpha(r)], \alpha(r)] = 0 \qquad for\ all\ r \in M. \qquad (19)$$

Linearization Eq. (19) gives

$$[[R(1), \alpha(r)], \alpha(y)] + [[R(1), \alpha(y)], \alpha(r)] = 0 \qquad for\ all\ r, y \in M. \qquad (20)$$

Putting ry for y in Eq. (20), because of Eq. (19) and Eq . (20), we obtain

$$0 = [[R(1), \alpha(r)], \alpha(ry)] + [[R(1), \alpha(ry)], \alpha(r)]$$

$$= \alpha(r)[[R(1), \alpha(r)], \alpha(y)] + [\alpha(r)[R(1), \alpha(y)], \alpha(r)] + [[R(1), \alpha(r)]\alpha(y), \alpha(r)].$$

Thus, we have

$$[R(1), \alpha(r)][\alpha(y), \alpha(r)] = 0 \qquad for\ all\ r, y \in M.$$

Since α is a surjective endomorphism of M. The substitution $\alpha(y)R(1)$ for $\alpha(y)$ in the above relation gives

$$[R(1), \alpha(r)]\alpha(y)[R(1), \alpha(r)] = 0 \qquad for\ all\ r, y \in M.$$

whence it follows $R(1) \in Z(M)$, which reduces Eq. (18) to the form

$$R(r) = R(1)r \qquad for\ all\ r \in M.$$

The proof of the theorem is complete.

Corollary 3.10

Let M be a 2-torsion free semiprime semiring with an identity element and let $R: M \to M$ be an additive mapping. Suppose that $R(r^3) + rR(r)r' = 0$, holds for all $r \in M$. In this case R is a centralizer.

4. Conclusions

In this work, we discussed an inverse semiring, centralizer of inverse semiring, Jordan centralizer, α-centralizer, Jordan α-centralizer. And then we prove that every left (right) Jordan α-centralizer is a left (right) α-centralizer.

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