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Modified Model to Calculate Low Earth Orbit (LEO) for A satellite with Atmospheric Drag

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Abstract:
In this paper, the satellite in low Earth orbit (LEO) with atmospheric drag perturbation have been studied, where Newton Raphson method to solve Kepler equation for elliptical orbit (i=63°, e = 0.1and 0.5, Ω =30°, ω =100°**)** using a new modified model. Equation of motion solved using 4th order Rang Kutta method to determine the position and velocity component which were used to calculate new orbital elements after time step (Δt) for heights (100, 200, 500 km) with (A/m) =0.00566 m^2/kg. The results showed that all orbital elements are varies with time, where (a, e, ω, Ω) are increased while (i and M) are decreased its values during 100 rotations.The satellite will fall to earth faster at the lower height and width using big values for eccentricity (e) and (A/m) ratio. Were the results got lifetime at height = 200, e = 0.1 equal 11 days for A/m =0.02 m2/kg, while satellite's lifetime at height = 200, e = 0.1 for A/m = 0.09 m2/kg equal 6 days.

Keywords: orbital elements, perturbation, satellite, atmospheric drag.

نموذج مطور لحساب المدار الواطئ للقمر الصناعي بوجود كبح الغلاف الجوي

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الخلاصه:
تم في هذا البحث في دراسة المدار المنخفض للقمر الصناعي بوجود اضطراب كبح الغلاف الجوي ، حيث استخدمت طريقة نيوتن رافسن لحل معادله كبلر لمدار القطع الناقص لـi=63°، (e = 0.1، 0.5) ، Ω=30° ω=100° باستخدام نموذج مطور لحساب تغير العناصرالمدارية بوجود الاضطراب. تم حل معادلة الحركة باستخدام طريقة رانج كوتا للمرتبة الرابعة لحساب مركبات الموقع والسرعه للقمر الصناعي التي تستخدم لحساب العناصر المدارية الجديده بعد زمن (Δt) للارتفاعات 100،200، (500 km)، A/m = 0.00566 m^2/kg. النتائج بينت ان كل العناصر المدارية تتغير مع الزمن، حيث ان العناصر (a، e، ω، Ω) تتزايد قيمها بينما العناصر (i، M) تتناقص قيمها خلال 100 دورة للقمر الصناعي. القمر الصناعي سيسقط نحو الارض خلال وقت اقصر عند المدارات الواطئة ولقيم A/m وللشذوذ (e) الكبيرة حيث ان عُمر القمر الصناعي 11 يوم لـ A/m= 0.02 m^2/kg، e =0.1 ، بينما يكون عُمر القمر الصناعي يساوي 6 ايام لـ A/m = 0.09 m^2/kg ، e =0.1 .

Introduction:

A satellite is an artificial object, which has been placed into special orbit around the Earth. Such objects are sometimes called artificial satellites to distinguish them from natural satellites such as the Moon. [1]

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The total number of artificial satellites orbiting the Earth until 2013 is around 8300 of these, about 3000 are not operational having lived out their useful life and are part of the space debris.[2,3]

Satellites classification according to its properties as (Astronomical satellites, Biosatellites, Navigational satellites, Communications satellites and Weather satellites[4, 5].

The orbits can be classified: according its altitude as (High, Medium and Low earth orbit) [6-8], according its Inclination as (Inclined orbit, Polar orbit and Polar sun synchronous orbit) [9, 10], and according its Eccentricity as (Circular orbit, Elliptic orbit, Hyperbolic orbit, Parabolic orbit, Escape orbit and Capture orbit) [11,12].

The satellite orbits are affected by many perturbations like (oblateness, solar radiation pressure, other body attraction and Earth atmospheric drag), in the LEO orbits the effect of the atmospheric drag and oblateness of the earth is much more than other perturbations [11, 13]. Then in this research will discuss the perturbations on low earth orbit include atmospheric drag only.

Theory:

In celestial mechanics, an elliptic orbit is a Kepler orbit with the eccentricity less than 1, this includes the special case of a circular orbit with eccentricity equal to zero. There are six elements for elliptical orbit, dimensional elements and other for rotation elements as a following. [9]

Semi major axis (a): which is define the size of the orbit. Eccentricity (e): Define the shape of the orbit, describing how flattened it is compared with a circle. Inclination (i): measure in degree ($0^o < i < 180^o$). Mean anomaly (M), argument of perigee (ω) and the right ascension of ascending node (Ω): are expressed as an angle, ($0^o < \Omega < 360^o$) [13,14].

To obtain the position and velocity of the satellite at specified time t, must be that the semi major axis and time of perigee passage were got it later to calculate the mean anomaly M and then find the eccentric anomaly value E [13].

The mean motion n can be written as equation (1) [15],

$$n = \frac{2\pi}{T} = K\sqrt{\frac{\mu}{a^3}} \qquad (1)$$

Where K= 0.07436616 $(e.r)^{3/2}/min$ for artificial satellite motion

At any time the mean anomaly used to describing the location of the satellite in an orbit:[16]

$$M = n(t-\tau) \qquad (2)$$

Or, $M = M + step$ (3)

Where τ is time of perigee passage. [15,17- 19]

The eccentric anomaly for the orbit calculated as [14, 15]

$$E = M + e_o \sin E \qquad (4)$$

Generally the equation (4) is Kepler's equation, where e_o in degrees.

Where: eccentricity (e) and mean anomaly (M) are given at beginning to solve equation (4). There are some methods for calculate E then the value of eccentric anomaly E using then to calculate true anomaly (f), We adopted the Newton-Raphson method in solving a Kepler's equation :

If $$f(E_i) = E_i - e_o \sin E_i - M \qquad (5)$$

Then $$f'(E_i) = 1 - e_o \cos E_i \qquad (6)$$

Apply Newton-Raphson method base in approximately

$$\Delta E_i = -\frac{f(E_i)}{f'(E_i)} \qquad (7)$$

Determine a new (E) from

$$E_{i+1} = E_i + \Delta E_i \qquad (8)$$

Now to find the Cartesian coordinate (x_w, y_w, z_w) for the satellite in its orbit as the follow [20, 21]:

$$\left.\begin{aligned} x_w &= a(\cos E - e) \\ y_w &= a\sqrt{1-e^2}\sin E \\ z_w &= 0 \end{aligned}\right\} \qquad (9)$$

Where: $a = rp/(1-e)$; $rp = h + R_e$; R_e =3678.270 km ; e value was input at begin and then it measured by equation (22)

And the displacement radius r will be [22,23]

$$r = a(1 - e\cos E) \qquad (10)$$

By differentiation for (x_w, y_w, z_w) for a satellite in its orbit we get: [16]

$$\left.\begin{aligned}\dot{x}_w &= \frac{\sqrt{\mu a}}{r}\sin E\\ \dot{y}_w &= \frac{\sqrt{\mu a\,(1-e^2)}}{r}\cos E\\ \dot{z}_w &= 0\end{aligned}\right\} \qquad (11)$$

A velocity $\dot{r}$ can be writing as: [16]

$$\dot{r} = \frac{\sqrt{\mu a}}{r}\; e \sin E \qquad (12)$$

Then, by Gaussian vector (conversion matrix), convert the position and velocity of a satellite from the orbital plane to the Earth equatorial plane as:

Which content Euler angles (ω, i, Ω) [9, 16].

$$\text{for a position} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = R^{-1}\begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} \quad \text{, for a velocity} \quad \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = R^{-1}\begin{bmatrix} \dot{x}_w \\ \dot{y}_w \\ \dot{z}_w \end{bmatrix} \qquad (13)$$

Where R^{-1} is the inverse of Gauss matrix

$$R^{-1} = \begin{bmatrix} P_x & Q_x & W_x \\ P_y & Q_y & W_y \\ P_z & Q_z & W_z \end{bmatrix} \qquad (14)$$

Where:

$$\left.\begin{aligned}
P_x &= \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i\\
P_y &= \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i\\
P_z &= \sin\omega\sin i\\
Q_x &= -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i\\
Q_y &= -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i\\
Q_z &= \cos\omega\sin i\\
W_x &= \sin\Omega\sin i\\
W_y &= -\cos\Omega\sin i\\
W_z &= \cos i
\end{aligned}\right\}(15)$$

Thus the position components in the equatorial plain

$$\left.\begin{aligned}
x &= P_x x_w + Q_x y_w + W_x z_w\\
y &= P_y x_w + Q_y y_w + W_y z_w\\
z &= P_z x_w + Q_z y_w + W_z z_w
\end{aligned}\right\}(16)$$

$$r = (x^2 + y^2 + z^2)^{1/2} \qquad (17)$$

The elliptical orbit

The elliptical orbital elements in general are (a, e, i, Ω, ω, M) can be calculated from the components of position, velocity and angular momentum as follows:

The inclination (i) of the orbit from the equatorial plane is given by [16]:

$$\tan i = \frac{\sqrt{h_x^2 + h_y^2}}{h_z} \qquad (18)$$

The longitude of ascending node (Ω) is calculated as [16, 17]:

$$\tan \Omega = \frac{h_x}{-h_y} \qquad (19)$$

The argument of perigee (u) can be found as [16]:

$$\tan u = \frac{zh}{-xh_y + yh_x} \qquad (20)$$

The semi-major axis ($\grave{a}$) of the orbit calculated as:

$$da = -\left((n * a^2 * 9.87 * 2.2 * \left(\frac{A}{m}\right) * \rho)\right)$$

$$\grave{a} = a + da \qquad (21)$$

The eccentricity (e) of the orbit is calculated as [16, 18]:

$$e = \sqrt{1 - \frac{h^2}{\mu a}} \qquad (22)$$

The mean anomaly (M) is calculated as [18,19]:

$$M = E - \frac{x\dot{x} + y\dot{y} + z\dot{z}}{\sqrt{a\mu}}$$

Or $M = E - e * sin\, e$ (23)

The true anomaly (f) is calculated as [18- 21]:

$$\tan\frac{f}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2} \qquad (24)$$

Where the eccentric anomaly (E) can calculate as [18, 19]:

$$\tan E = \left(\frac{1-\frac{r}{a}}{x\dot{x} + y\dot{y} + z\dot{z}}\right)\sqrt{a\mu} \qquad (25)$$

Atmospheric drag acceleration

A satellite in low Earth orbit experiences some drag as it moves through the Earth's thin upper atmosphere. In time, the action of drag on a satellite will cause it to spiral back into the atmosphere, eventually to disintegrate or burn up. If a space vehicle comes within 120 to 160 km of the Earth's surface, atmospheric drag will bring it down in a few days, with final disintegration occurring at an altitude of about 80 km. [24]

The relation that describes the acceleration due to the atmospheric drag is: [25]

$$\vec{a}_{\text{Drag}} = -\frac{1}{2}\rho\frac{C_D A}{m}v_{rel}^2\frac{\vec{v}_{rel}}{\|\vec{v}_{rel}\|} \qquad (26)$$

The equation of motion with perturbation is:

$$\ddot{r} = \mu\frac{\partial r}{\partial t^2} + \vec{a}_{\text{Drag}}$$

There are many techniques to solve it, but we mention classical Runge-Kutta algorithm 4th order and has 4 stages. The stage number indicates how often the right hand function *f*(x, *t*) has to be evaluated [25]. Then using this coordinates$x, y, z, (\dot{x}, \dot{y}, \dot{z})$ to recalculate the orbital elements by equations (18-25)

The new state vector can be obtained by [25]

$$x_{i+1} = x_i + \frac{h_{step}}{6} * (k_1 + 2 * k_2 + 2 * k_3 + k_4) \qquad (27)$$

Where: h_{step} is step width in time, the four derivatives k_1 through k_4 are computed as following:

$$\left.\begin{aligned} k_1 &= f(x_i) \\ k_2 &= f(x_i + \frac{h_{step}}{2}\,.\,k_1) \\ k_3 &= f(x_i + \frac{h_{step}}{2}\,.\,k_2) \\ k_4 &= f\left(x_i + h_{step}\,.\,k_3\right) \end{aligned}\right\} \quad (28)$$

Lifetime of the a satellite

The lifetime is the time from the beginning to the destruction or escape to other type of orbit. It is strongly dependent on satellite altitude. The lifetime of satellites is very large for MEO and HEO, because the satellites become out of the largest force, the basic principle to know satellite's lifetime, for elliptical orbit only depends upon two orbital elements, the semi-major axis (a) and the eccentricity (e), because it indicates the size and shape of orbit [25]

Results and discussion:

Atmospheric drag acceleration was calculated by equation (26), the figures 1-6 showed that Atmospheric drag effect on satellite's orbital element at different orbital heights, A/m = 0.0056 (m^2/kg) for 1000 steps, then according results for orbital elements that produced by Atmospheric drag acceleration:

Semi major axis (a) is illustrated in figures 1.a, b, c, d, e, f showed clear linearly decreases in (a) value, where a satellite's altitude decreased about 50 times at h_p=100 km more than at h_p=200 km for (e = 0.1) during 100 rotations. The variation of (a) is very small at height value for r_p = < 500 this means the atmospheric drag is very small at is there.

Figure 2a to figure 2f shown inverse relationship for Eccentricity (e) with time at (e=0.1, 0.5), ($h_{p=}$ 100, 200, 500) km during 100 rotation which means the orbit closed after many orbit to circular orbit. The variation is very small for small values of (e) and not much affected by change height.

Inclination (i) illustrated in figures 3 a, b, c, d showed that a curvature increases while in (4.11.e, f) figures was just increases in (i) value at (e =0.1, 0.5), heights (h_p=100, 200, and 500 km), with time after 100 rotations.

The results for mean anomaly at epoch time (M) is shown from figure 4a to 4f are linearly increasing with time at (e = 0.1, 0.5), (h_p= 100, 200, 500 km).

Figures 5a to 5f display results Longitude of ascending node (Ω) showed concave down curve decreases in (Ω) values with time, after 100 rotations.

Argument of perigee (ω) displayed in figures 6.a, b, c, d, e, f showed linearly increases for (ω) value with time, during 100 rotations.

The figures 7a and 7b show the variation r_p with time under the effect of atmospheric drag, until the satellite is unable to make another revolution if we are not going reforms (Corrections) on orbit, where A/m ratio was studied and a result showed that lifetime for satellite = 11 days for A/m=0.02, while its = 6 day for A/m = 0.09 then a satellite will fall to Earth at r_p < 6380km.

In general the results get the fact that an orbit is more stable for less eccentricity and A/m ratio.

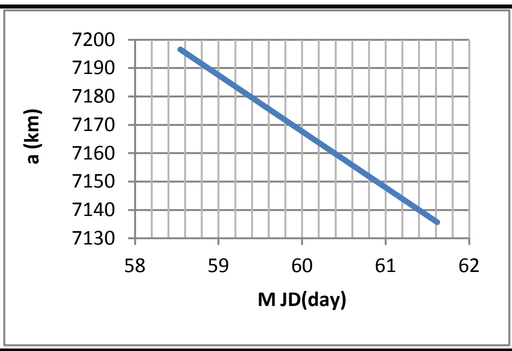

Figure 1a-Variation of semi-major axis with time due to Atmospheric drag at h_p=100 km, e=0.1.

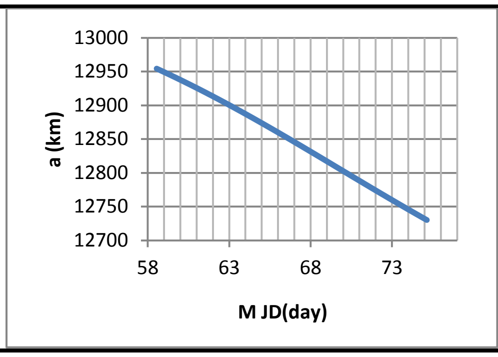

Figure 1b-Variation of semi-major axis with time due to Atmospheric drag at hp=100 km, e=0.5.

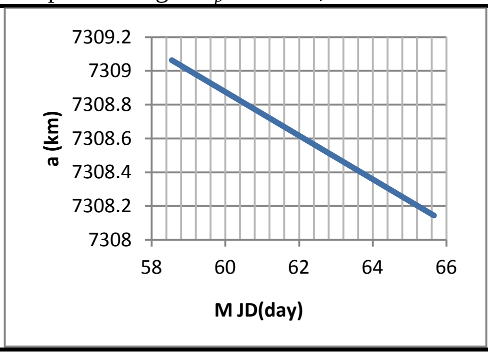

Figure 1c-Variation of semi-major axis with time due to Atmospheric drag at hp=200 km, e=0.1.

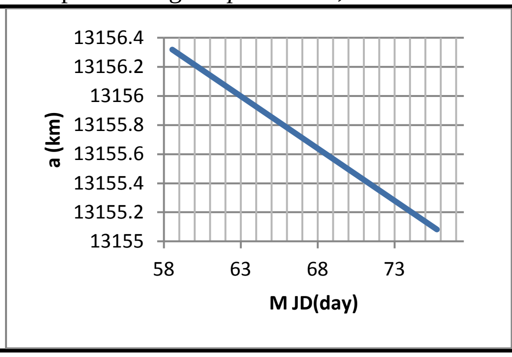

Figure 1d-Variation of semi-major axis with time due to Atmospheric drag at hp=200 km, e=0.5.

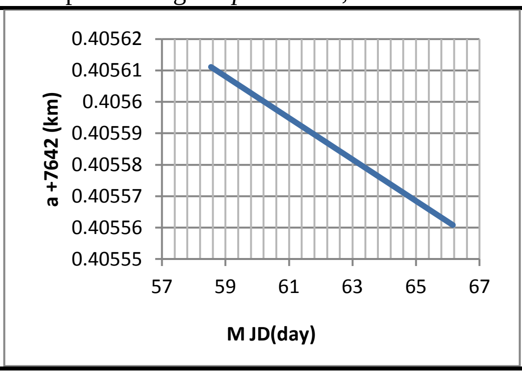

Figure 1e-Variation of semi-major axis due to Atmospheric drag at hp=500km, e=0.1.

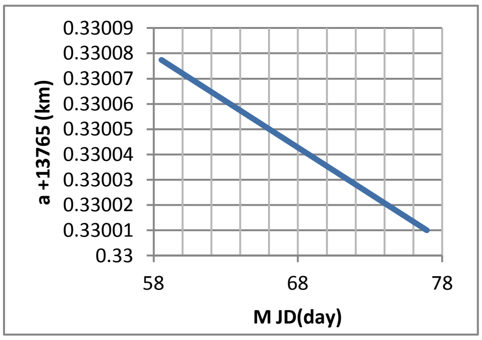

Figure 1f-Variation of semi-major axis with time due to Atmospheric drag at hp=500km, e=0.5.

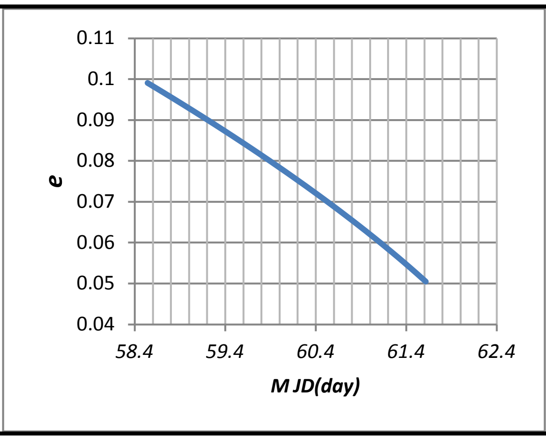

Figure 2a-Variation of eccentricity with time due to Atmospheric drag at *hp*=100km, *e*=0.1

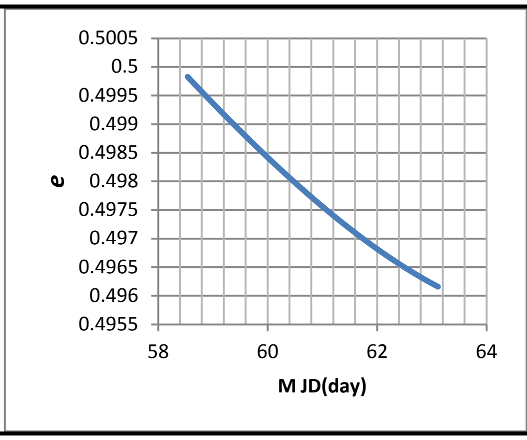

Figure 2b-Variation of eccentricity with time due to Atmospheric drag at *hp*=100km, *e*=0.5.

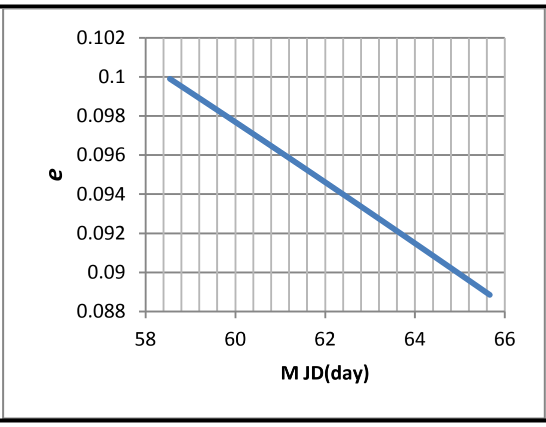

Figure 4.10.c-Variation of eccentricity with time due to Atmospheric drag at *hp*=200km, *e*=0.1

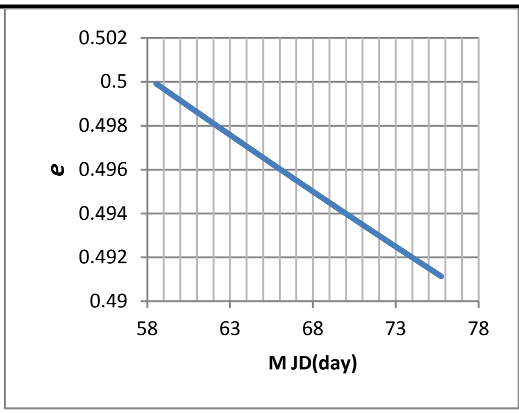

Figure 4.10.d-Variation of eccentricity with time due to Atmospheric drag at *hp*=200km, *e*=0.5.

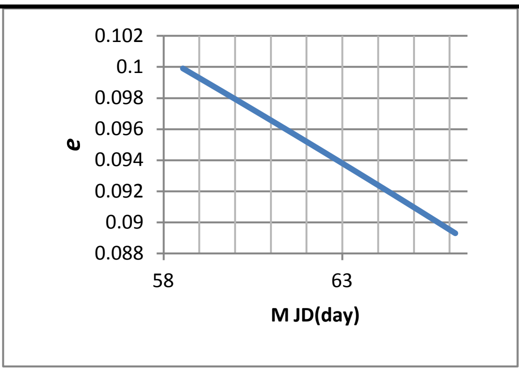

Figure 2.e-Variation of eccentricity with time due to Atmospheric drag at *hp*=500km, *e*=0.1

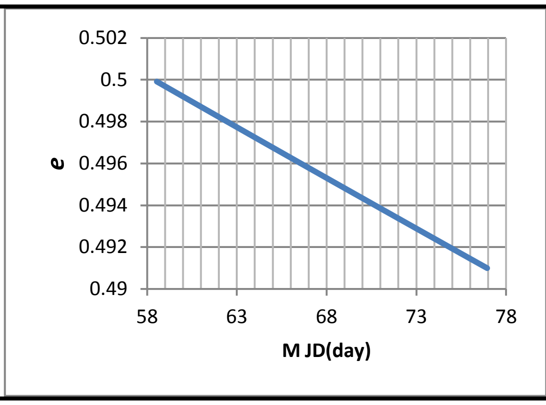

Figure 2.f-Variation of eccentricity with time due to Atmospheric drag at *hp*=500km, *e*=0.5.

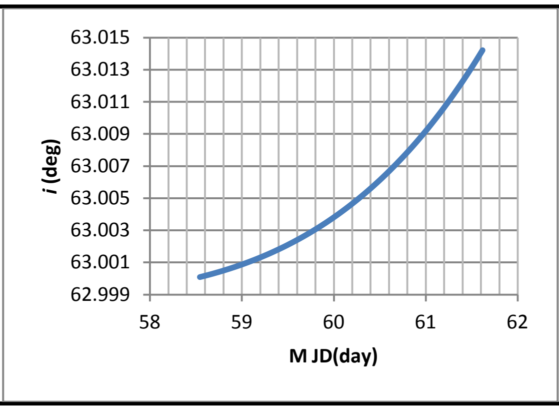

Figure 3.a-Variation of inclination due to Atmospheric drag at h_p= 100km, e=0.1.

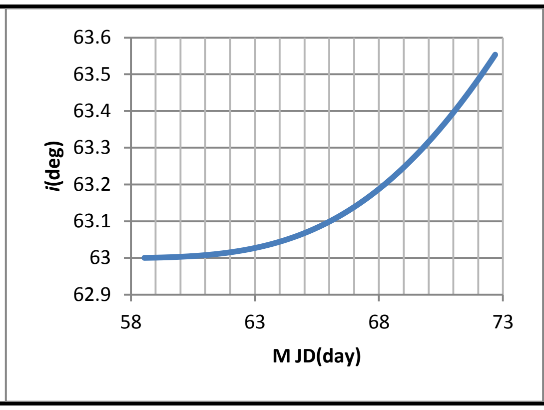

Figure 3.b-Variation of inclination with time due to Atmospheric drag at h_p= 100km, e=0.5.

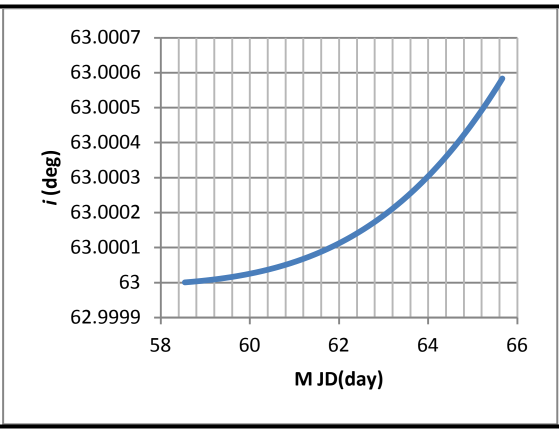

Figure 3.c-Variation of inclination due to Atmospheric drag at h_p= 200km, e=0.1.

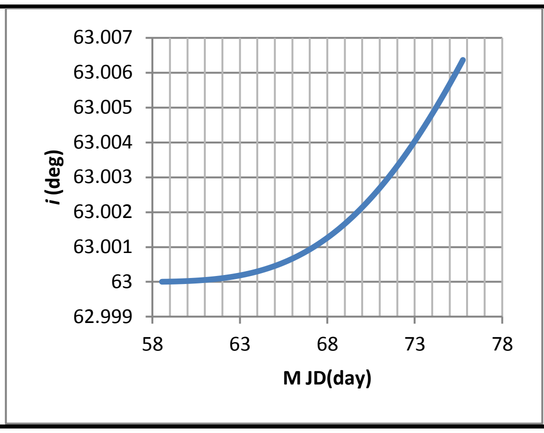

Figure 3.d-Variation of inclination with time due to Atmospheric drag at h_p= 200km, e=0.5.

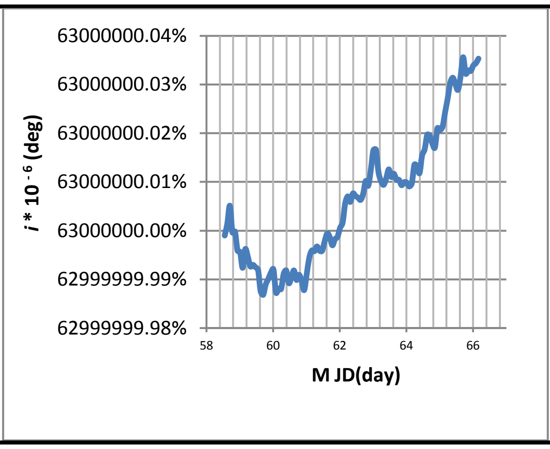

Figure 3.e-Variation of inclination with time due to Atmospheric drag at h_p= 500km, e=0.1.

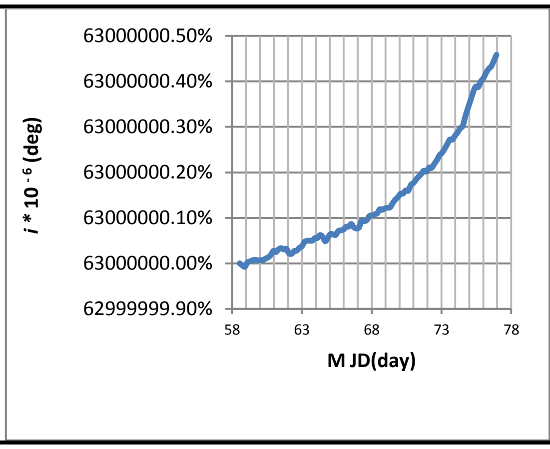

Figure 3.f-Variation of inclination with time due to Atmospheric drag at h_p= 500km, e=0.5.

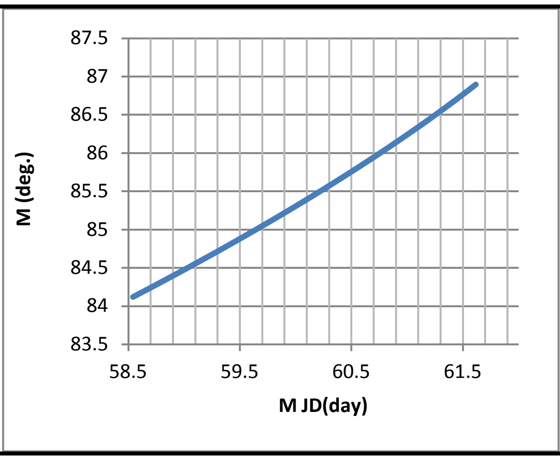

Figure 4.a-Variation of mean anomaly with time due to Atmospheric drag at h_p=100 km, e = 0.1.

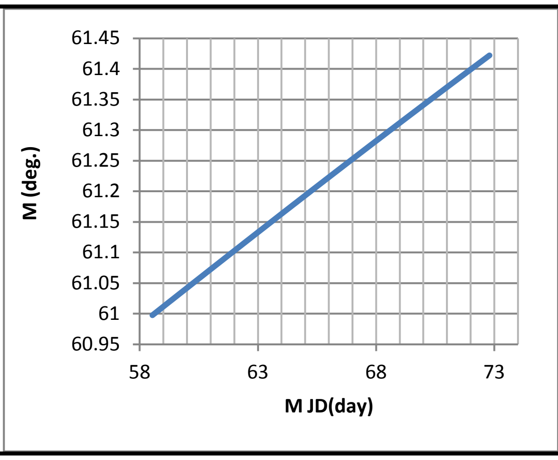

Figure 4.b-Variation of mean anomaly with time due to Atmospheric drag at h_p=100 km, e = 0.5.

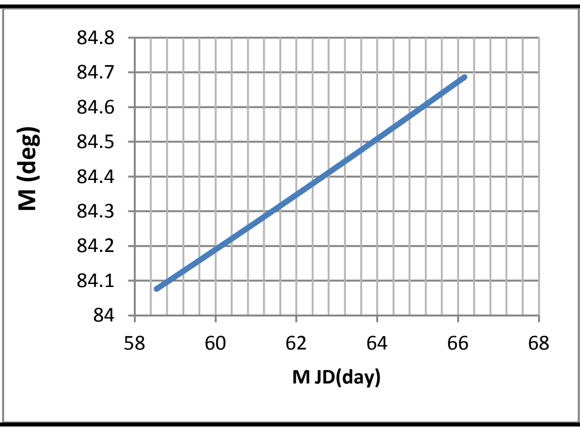

Figure 4.c-Variation of mean anomaly with time due to Atmospheric drag at h_p=200 km, e = 0.1.

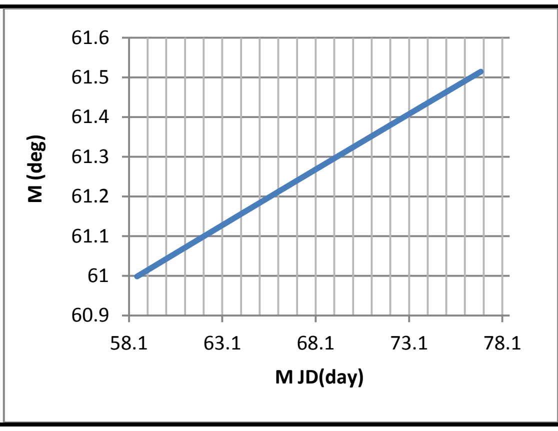

Figure 4.d-Variation of mean anomaly with time due to Atmospheric drag at h_p=200 km, e = 0.5.

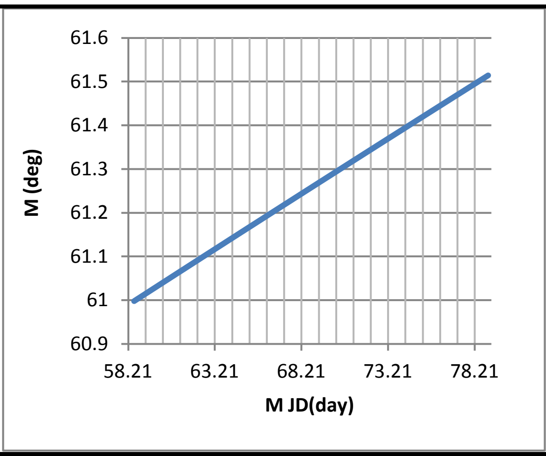

Figure 4.f-Variation of mean anomaly with time due to Atmospheric drag at h_p=500 km, e = 0.1.

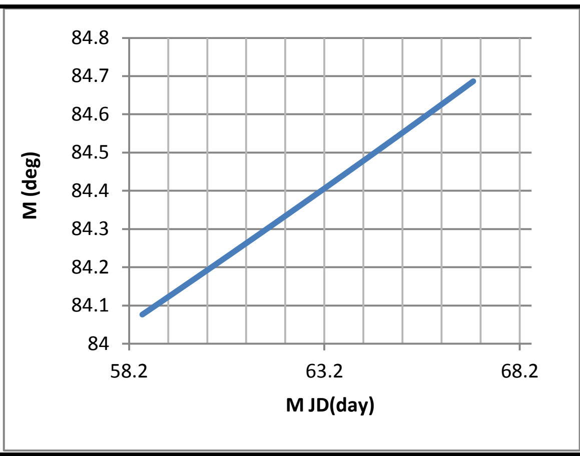

Figure 4.e-Variation of mean anomaly with time due to Atmospheric drag at h_p=500 km, e = 0.1.

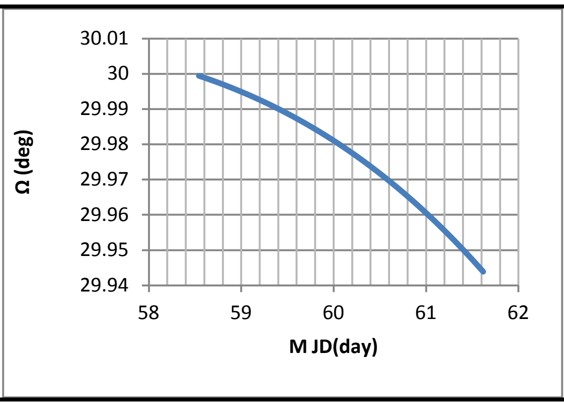

Figure 5.a-Variation of longitude of ascending node with time due to Atmospheric drag at h_p=100km, e = 0.1.

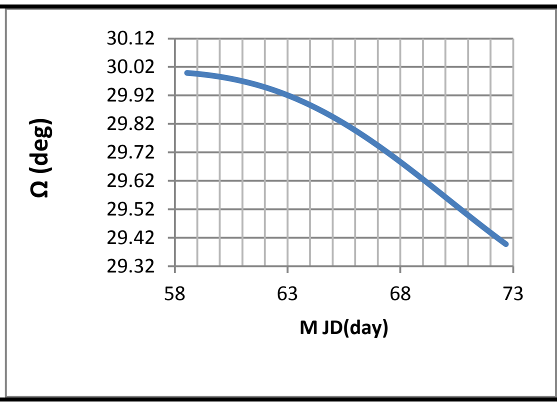

Figure 5.b-Variation of longitude of ascending node with time due to Atmospheric drag at h_p=100km, e = 0.5.

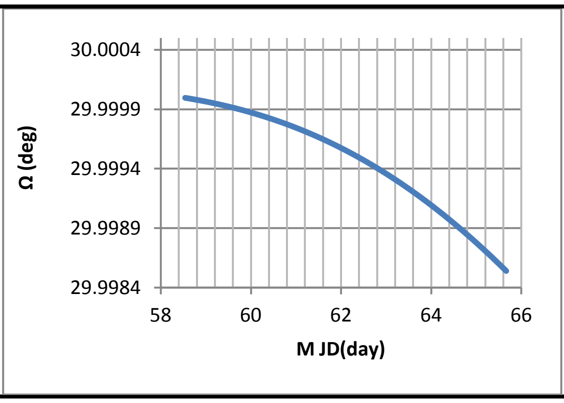

Figure 5.c-Variation of longitude of ascending node with time due to Atmospheric drag at h_p=200km, e = 0.1.

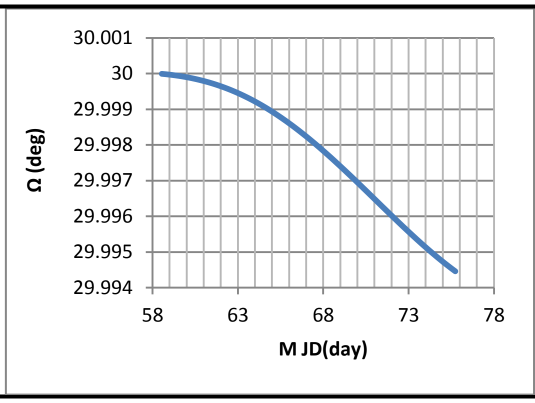

Figure 5.d-Variation of longitude of ascending node with time due to Atmospheric drag at h_p=200km, e = 0.5.

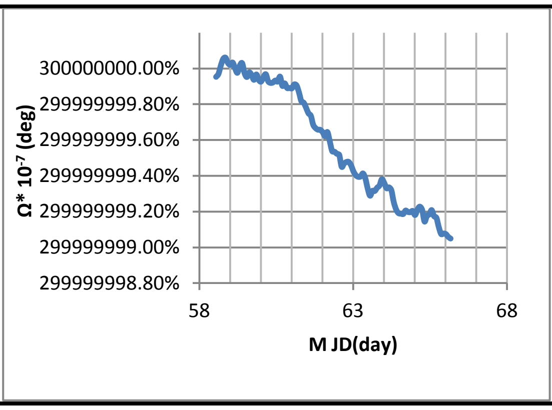

Figure 5.e-Variation of longitude of ascending node with time due to Atmospheric drag at h_p=500km, e = 0.1.

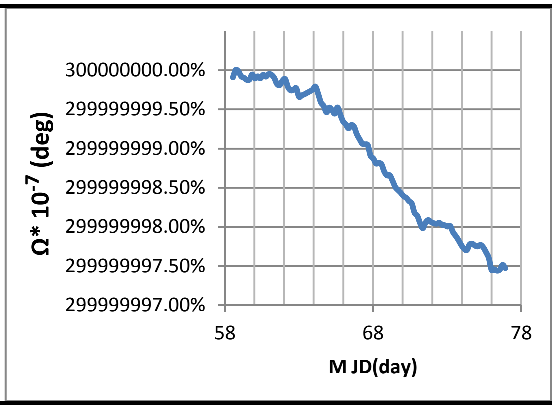

Figure 5.f-Variation of longitude of ascending node with time due to Atmospheric drag at h_p=500km, e = 0.5.

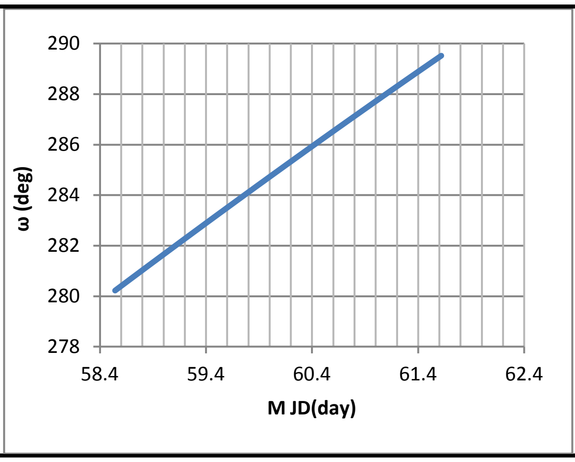

Figure 6.a-Variation of argument of perigee with time due to Atmospheric drag at h_p=100 km, e = 0.1.

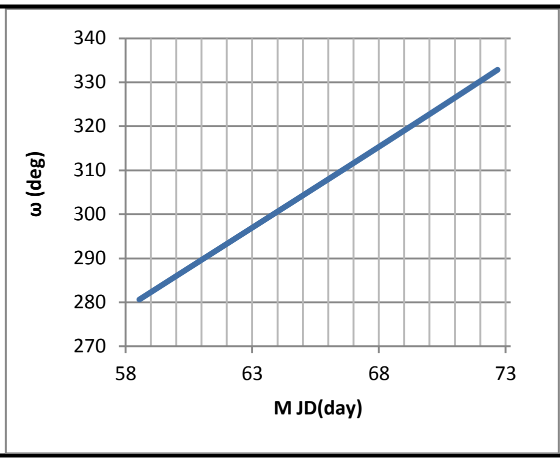

Figure 6.b-Variation of argument of perigee with time due to Atmospheric drag at h_p=100 km, e = 0.5.

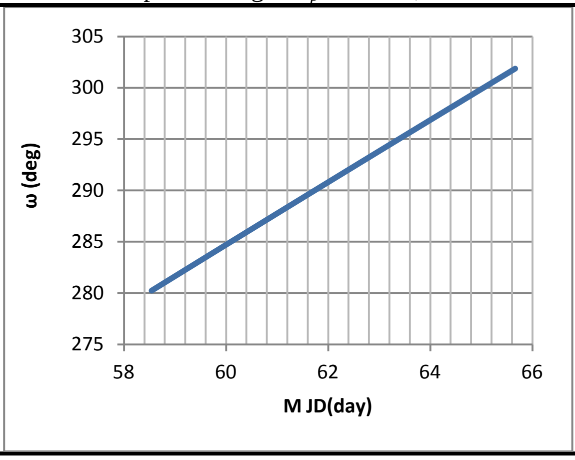

Figure 6.c-Variation of argument of perigee with time due to Atmospheric drag at h_p=100 km, e = 0.5.

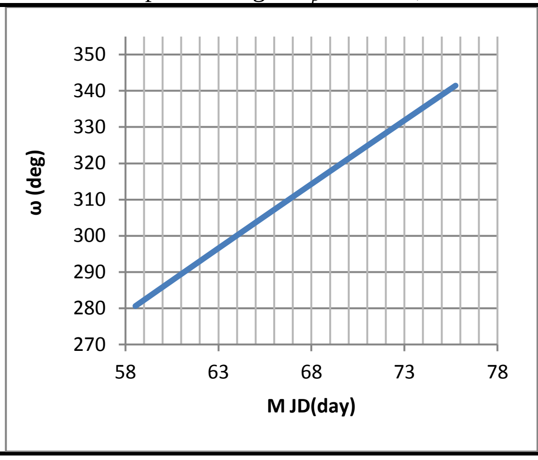

Figure 6d-Variation of argument of perigee with time due to Atmospheric drag at h_p=200 km, e = 0.5.

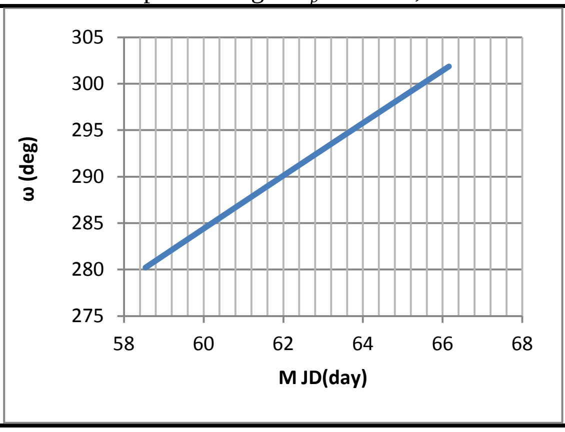

Figure 6.e-Variation of argument of perigee with time due to Atmospheric drag at h_p=500 km, e = 0.1.

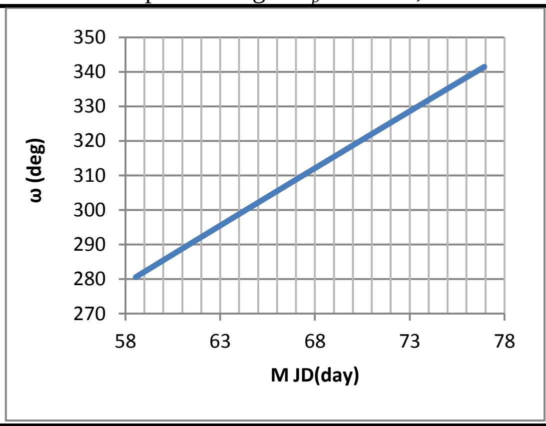

Figure 6.f-Variation of argument of perigee with time due to Atmospheric drag at h_p=500 km, e = 0.5.

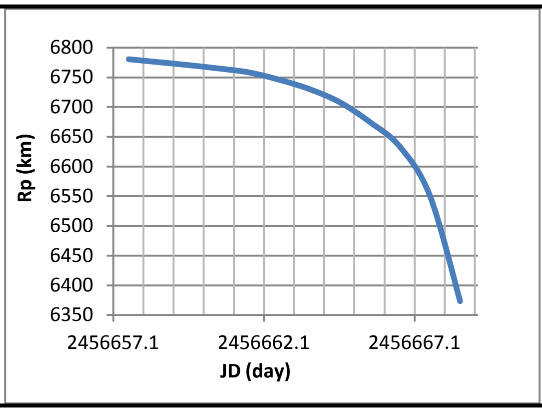

Figure 7.a-Show satellite's lifetime with atmospheric drag acceleration at h_p=200 km, e = 0.1, A/m =0.02 (m^2/kg).

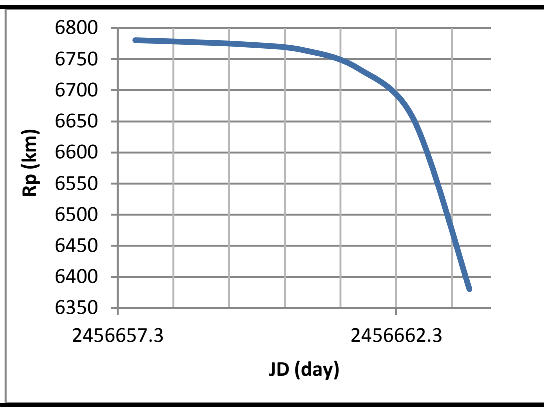

Figure 7.b-Show satellite's lifetime with atmospheric drag acceleration at h_p=200 km, e = 0.1, A/m =0.09 (m^2/kg).

Conclusion

1. The satellite low orbit was affected by many types of perturbations; the important one is the atmospheric drag.
2. The semi major axis and eccentricity (e) are reduced very slowly with bigger values of r_p.
3. The lifetime of the orbit is directly proportional with r_p and a, while it's inversely proportional with e.
4. The results for the modified model have good agreement with some references as (13, 23).

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