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# Decision Making Methods in Comparative Studies of Complex Economic Processes Management

**Mateusz Piwowarski*[1], Kesra Nermend[1], Mariusz Borawski[2]**
[1]Department of Computational Methods in Experimental Economics, Faculty of Economics and Management, Institute of IT in Management, University of Szczecin, Mickiewicza 64, Szczecin 71-101, Poland
[2]Department of Multimedia Systems, Faculty of Computer Science and Information Technology, West Pomeranian University of Technology Szczecin, Żołnierska 49, 71-210 Szczecin, Poland

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**Abstract**
In this study different methodological approaches are used and described by many features (indicators) of complex socio-economic process. Outcome of analysis has the most reliable and acceptable representation of the studied process specific to chosen case. In order to solve problems in this area (depending on the situation, case under consideration), methods from two groups are most often used: multidimensional comparative analysis and multi-criteria decision analysis. The first of these cases concern problems at the macro level (socio-economic development, demographic situation, population's living standards, etc.), in which the decision-maker's participation is relatively small (eg the selection of diagnostic variables or expert assessment). The second of these groups include issues in which the decision-maker's participation is significant which are subjective to the decisions taken and reflects his or her preferences. Among the decision support methods, one can also distinguish those that have both the characteristics of methods from the area of multidimensional comparative analysis and multi-criteria decision analysis. The article presents the combination of both trends exposing maximum possibilities of using selected methods used in the decision making by Polish schools. The general methodological assumptions, advantages of having approaches discussed (in relation to other known methods) as well as the applied aspects (exemplary applications) also presented.

**Keywords:** multidimensional comparative analysis • multi-criteria analysis methods • VMCM • PVM

## 1. Introduction

Phenomena those are not directly measured called complex economic phenomena (socio-economic development, demographic situation, standard of living, etc.) are often analyzed [1,2] in economic studies. Studied phenomenon have been presented with the most reliable and acceptable result based on the same actions of complex economic phenomena, Methods of multidimensional comparative analysis (MCA) are often applied [3,4,5] with multidimensional comparative analysis, where It can have the possibility of an objective and automated analysis, and solution of the problem under consideration. Here decision-makers have a very limited to the minimum (the decision maker is some way unknown or collective) participation and obtaining detailed piece of information about his preferences is impossible or uneconomic in terms of time and cost.

Methods used in this case to support decisions by appropriate processing and automatic data ordering also allows for 'objectification' (as far as possible) of the obtained rankings in most cases. Hence, the "objectification" of the ranking is applied to have the consistent ranking with the preferences of a majority or a group of 'typical' decision makers (by default rational and with

*Email: mateusz.piwowarski@usz.edu.pl

simplified preference structures). This group of methods include, e.g. TOPSIS [6,7,8], VIKOR [9,10], HELWIG [11], VMCM [12; 13] and PVM [12,14].

Certain economic decisions are more individualized (subjective) require decision maker participation to represent his or her preferences. These decision-making situations are considered in the context of Multiple Criteria Decision Analysis [13, 15]. The procedure of multi-criteria decision analysis (derived from operational research) including many evaluation criteria is characterized by a different methodological approach in the case of multidimensional comparative analysis.

The approach using multi-criteria analysis methods assumes a strong collaboration with the decision-maker and the possibly precise identification of his own preferences to arrange group variants. The results obtained in this way (recommendations for decisions) are strongly subjective and universality can be assumed, because they refer only to previously identified own and subjective preferences and priorities of a decision maker or a group of supported decision-makers [16, 17]. Therefore, the point is not to obtain an objective status of a case, rather to model the decision-making situation in accordance with the decision-maker's value system and to solve the problem.

Multiple Criteria Decision Analysis (MCDA) / Multiple Criteria Decision Making (MCDM) methods can be grouped as different criteria [6, 18, 19], e.g.:

– methods based on the outranking relation - ELECTRE methods [20, 21], PROMETHEE methods [22,23], ORESTE [24], REGIME [25], TACTIC [26], NAIADE [27], MELCHIOR [28], etc.;
– methods based on the utility function - AHP [29,30], ANP [31], DEMATEL [2], MAUT, REMBRANDT, MACBETH [1], SMART [32], UTA [33], etc.

Summing up the discussion on decision-making support in the aspect of multi-criteria and multidimensionality, one can point to some similarities as well as conceptual and methodological differences between the two approaches. In addition to the differences in the terminology used in multidimensional comparative analysis and multi-criteria decision analysis, there are different applications of methods from these two areas, but the scope of decision-maker participation in the decision process is noteworthy. In multi-criteria decision support methods, the participation of a decision-maker at various stages of the process is much larger, hence assessment is subjective and the decision-making process is difficult to automate. However, a group of methods (e.g. TOPSIS, VIKOR, VMCM, PVM) that has common characters of methods from the area of multidimensional comparative analysis and multi-criteria decision analysis can be distinguished.

The essence of building these methods is limiting the necessary to minimum interaction for decision maker. For example, in the PVM method, the presence of a decision maker does not have to be large (a feature of multidimensional comparative analysis) or it may be significant (a feature of multi-criteria decision analysis) [34]. In the first case, the decision-maker specifies own preferences by defining the criteria and assigning them with a character, then a motivating and demotivating preference vector is calculated (for motivating and demotivating criteria). Based on these vectors, the criteria weights are automatically determined and an artificial preference vector is calculated. This approach can be considered as an objective solution to the decision problem because of the limited participation of the decision-maker. In the second case, the decision maker specifies own preferences defining the criteria assigning characters to these criteria with determining weight of the criteria and gives the values of criteria in the form of two vectors (the motivating and the demotivating preferences vector). Based on these vectors and decision-maker's preferences the preference vector is calculated. Being decision maker's participation much larger in this approach makes the evaluation more subjective [12].

The aim of the article is to present the possibilities using of the authors methods (VMCM, PVM), having both the features of methods from the area of Multidimensional Comparative Analysis (MCA) and Multiple Criteria Decision Analysis (MCDA). General methodological assumptions, advantages of the approaches discussed (in relation to other known methods) as well as applied aspects will be presented. The reference to the results of previous own studies (e.g. fatigue when making decisions) shows the superiority of the proposed method (PVM) over others.

## 2. Materials and Methods

### 2.1. Experiments

The research experiment concerned the use of two VMCM (Vector Measure Construction Method) and PVM (Preference Vector Method) methods, having both features of the MCA and MCDA methods to solve two problems.

The first study concerned the issue of choosing a European country that offers the best conditions for researchers. Two scenarios were developed for this study. In the first scenario (based on the adopted criteria), a ranking of countries with according to the so-called artificial pattern was made. Then, taking into account the decision-maker's preferences, two real objects were selected: a pattern and anti-pattern. In our case, the best country in the ranking became the pattern. The anti-pattern was the last country in the ranking. The purpose of this research concept was to enable the general ranking of countries and the possibility of making comparison with respect to the selected real object (country), which was the pattern. VMCM method was applied as a methodical apparatus.

The indicators (criteria) used in this study were:

$x_1$ – Level of internet access - households;

$x_2$ – Research and development expenditure, by sectors of performance;

$x_3$ – Employment in high- and medium-high technology manufacturing sectors and knowledge-intensive service sectors;

$x_4$ – Human resources in science and technology;

$x_5$ – % of GDP Government sector Research and development expenditure, by sectors of performance.

Analyzed countries (variants) are Austria (AT), Belgium (BE), Bulgaria (BG), Cyprus (CY), Czechia (CZ), Germany (DE), Denmark (DK), Estonia (EE), Greece (EL), Spain (ES), Finland (FI), France (FR), Croatia (HR), Hungary (HU), Ireland (IE), Iceland (IS), Italy (IT), Lithuania (LT), Luxembourg (LU), Latvia (LV), Macedonia (MK), Malta (MT), Netherlands (NL), Norway (NO), Poland (PL), Portugal (PT), Romania (RO), Serbia (RS), Sweden (SE), Slovenia (SI), Slovakia (SK), Turkey (TR), United Kingdom (UK). Data referred to the year 2017 have taken from the Eurostat database.

The second study concerned the issue of choosing a European country, which is the most-friendly for living for seniors (retired). In contrast to the first study, the individual preferences of a decision maker are important here. Usually multiple criteria methods are used in such situations (e.g. AHP, ANP, REMBRANDT, etc.). In this type of method, the ranking is usually created based on the pairwise comparison procedure. The creators of these methods recommend a small number of considered criteria and decision alternatives. Therefore, authors of this article propose to apply PVM method, which can solve the problem of a larger number of criteria and decision options. At the same time, this method allows individual assessment of alternatives according to their preferences. The study comprised of 2 stages (according to PVM procedure). In the first stage, a general ranking of European countries was made, taking into account only quantitative criteria ($x_1$-$x_4$). In the second stage, the countries from the top of the ranking were selected (according to the PVM procedure) and then the objects (countries) were compared on the basis of the additional quality criterion, which was 'climate'. Further, the final ranking was made, which included both the quantitative criteria and the 'climate' quality criterion.

The indicators (criteria) used in this study were:

$x_1$ – At-risk-of-poverty rate of older people;

$x_2$ – Median relative income of elderly people (60+);

$x_3$ – Individuals using the internet for interaction with public authorities;

$x_4$ – 4. Social benefits other than social transfers in kind, payable, Percentage of gross domestic product (GDP);

$x_5$ – Climate.

The considered decision alternatives (countries) were the same as in the first study, except for Macedonia, Serbia, Turkey (no data available).

**2.2. Vector Measure Construction Method (VMCM)**

The VMCM utilizes the vector calculus properties in order to build vector synthetic measure (basing on the definition of the scalar product) [24]. Such an approach allow for making raking, classification of objects and study of change dynamics. The procedure of VMCM comprises 8 stages [12]:

Stage 1. Selection of variables.

Stage 2. Elimination of variables.

Stage 3. Defining the diagnostic variables character.

Stage 4. Assigning weights to diagnostic variables.

Stage 5. Normalization of variables.

Stage 6.Determination of the pattern and anti-pattern.
Stage 7. Building the synthetic measure.
Stage 8. Classification of objects.
*Stage 1. Selection of variables*
Selection of variables can be implemented in two ways, using the approach related to substantive and logical selection and the approach related to the elimination of variables such as those characterized by a high degree of collinearity.
*Stage 2. Elimination of variables*
Elimination of variables is carried out by using significance coefficient of features:

$$V_{x_i} = \frac{\sigma_i}{\bar{x}_i} \tag{1}$$

where:

$x_i$ – $i$ -th variable,

$\sigma_i$ – standard deviation of the $i$ -th variable,

$\bar{x}_i$ – mean value of the $i$ -th variable,

whereas:

$$\bar{x}_i = \frac{\sum_{j=1}^{n} x_{i_j}}{n} \tag{2}$$

and:

$$\sigma_i = \sqrt{\frac{\sum_{j=1}^{n}\left(x_{i_j} - \bar{x}_i\right)^2}{n-1}} \tag{3}$$

Variables for which significance coefficient values are within the range $\langle 0; 0{,}1 \rangle$, are quasi-constant and such variables should be eliminated from the set of variables under consideration [25].
*Stage 3. Defining the diagnostic variables character*
Definition of the diagnostic variables character refers to assigning diagnostic variables to the one of three groups are stimulants, destimulants and nominants. Stimulants are variables having greater values mean the higher level of development of studied phenomena, e.g. considering the quality of life there will be: number of GPs, cars, residential area per person, etc. Destimulants are variables having smaller values mean the higher level of development, for instance, considering the standard of living there will be: inflation, unemployment, etc. While nominants are variables having desired values are within a specific range (e.g. natural growth, lending rate, etc.).
*Stage 4. Assigning weights to diagnostic variables*
Weight systems are constructed in order to take into account the different impact of diagnostic variables on aggregate measures. Two approaches apply when determining variable weights. The first uses non-statistical information where weights are usually determined by the expert assessment method called substantive weights. The second approach is related to the use of information from various types of statistical materials takes statistical weights. In practice, we often set weights using the second approach based on statistical resources.

Weighing variables can be carried out for example with the use of a relative information value meter (other measures may also be used):

$$w_i = \frac{V_{x_i}}{\sum_{k=1}^{m} V_{x_k}} \tag{4}$$

where $w_i$ means variable's weight .
*Stage 5. Normalization of variables*
Variables used in studies are heterogeneous because they describe the various properties of objects. They can occur in different units of measure additionally hinder any arithmetic operation. Therefore

the next stage in the construction of the measure of development that must be carried out consists in normalizing variables. This process leads not only to the elimination of units of measurement, but also to equalize the values of variables. Standardization is the most commonly used normalization techniques.

*Stage 6. Determination of pattern and anti-pattern*

Pattern and anti-pattern can be selected as real-life objects. It is also possible to determine automatically the pattern and anti-pattern based on the first and third quartiles. Where coordinates of the pattern accordingly are taken for stimulants values of the third quartile and for destimulants values of the first quartile. In case of anti-pattern the procedure is inversed as values in the first quartile are anti-pattern coordinates for stimulants and values in the third quartile for destimulants.

*Stage 7. Construction of the synthetic measure*

Values of variables of the examined objects in the vector space are interpreted as vector coordinates. Each object therefore determines a specific direction in space. The pattern and anti-pattern difference is also a vector which determines a certain direction in space. Along this direction, the aggregate measure value for each object is calculated. This difference can be treated as a monodimensional coordinate system where coordinates are calculated based on the formula [35, 36].

*Stage 8. Classification of objects*

Values of the aggregate measure allow for ranking objects, thus it is possible to determine which of them are "better" and which are "worse". They also allow determining which are similar to each other in terms of adopted criteria. In order to better visualize the results of calculations, objects can be divided into classes with similar measurement values. This is particularly important in the case of spatial objects. Such a ranking can be presented in the form of a map where individual objects are visualized using the colors assigned to individual classes.

**2.3. Preference Vector Method (PVM)**

The Preference Vector Method constitutes development of the Vector Measure Construction Method and is a tool for solving multi-criteria decision-making problems. When solving decision-making problems by means of the PVM the decision-maker can express own preferences in a variety of ways.

In the majority of variants the PVM research process runs in ten stages [12]:

Stage 1. Formulating a decision problem.
Stage 2. Defining decision variants.
Stage 3. Setting evaluation criteria for decision variants.
Stage 4. Defining the criterion's character.
Stage 5. Assigning weight to criteria.
Stage 6. Standardizing the criterion values.
Stage 7. Determining the preferences vector.
Stage 8. Making the ranking (based on importance factor).
Stage 9. Selection of a decision variant (solution).
Stage 10. Assessment of effects of decision variant implementation.

*Stages 1-2. Formulating a decision problem and defining decision variants.*

These stages of the calculating procedure in PVM are the same as in the majority of multi-criteria methods of decision-making.

*Stage 3. Setting evaluation criteria for decision variants*

The selection of criteria is determined by decision-maker's preferences and depends on decision-making situation. For many such situations we are able to define a permanent set of criteria which can be used in similar cases. Based on the defined criteria a set of criterion values for considered decision variants is created. However, not always numerical values can be assigned to all criteria. The qualitative (subjective) criteria can be an example. The procedure for dealing with such criteria will be described in the last stage of PVM.

*Stage 4. Defining the criterion's character*

Typically, the decision-makers are not able to quantify their preferences, but they can indicate which decision variants are acceptable or unacceptable for them. Therefore the proposed criterion character should correspond to their intuition. The following classification of criteria: desirable, non-desirable, motivating, demotivating and neutral which form the criteria sets [37]. At this stage, specific characteristics should be assigned to every criterion. Each of criteria can be attributed with the one of the above mentioned characters.

*Stage 5. Assigning weight to criteria*

In the process of decision support, the importance of criteria comes from the subjective assessment of the situation by a decision-maker. For example, one criterion may be much more important than the others. There are different systems for determining weights for criteria. Their selection depends on the decision-maker with the purpose and scope of the study. For example we can assume that the weights of criteria $w_i$ are equal to 1. In a situation when it is necessary to add the importance of certain criteria, we can increase or reduce the weights. Once we have determined the weights, they should be normalized in order to eliminate the scales of values in which they have been expressed. Another, better and more preferred method of determining weights is to determine them based on pairwise comparisons used in the AHP method [29].

*Stage 6. Standardizing the criterion values*

Based on the principle criteria which describe decision variants are usually heterogeneous because they define different parameters of variants which are expressed in different measurement units and have different scales of value. Consequently, the data that are recorded in this way are incomparable. Therefore, we have to bring them to a form in which they can be compared. Hence the next stage of the PVM is the normalization of the criteria by which the measurement units are eliminated and the scales of the criterion values in studies are brought roughly to the same level. In the situation where a decision-maker provides only the values of the motivating preference vector and not able to determine own demotivating criteria in the form of a demotivating preference vector, it is recommended to use standard means to normalize the decision variants.

*Stage 7. Determining the preferences vector*

The preference vector represents the decision-maker's requirements with regard to the analyzed decision variants. It is a vector whose coordinates are made of values of the criteria calculated on the criterion's character. The criterion values of motivating or demotivating character. For criteria of motivating or demotivating character, the coordinates of this vector are the criteria values calculated on the basis of the difference between motivating $\vec{\Psi}$ and demotivating $\vec{\Phi}$ vector of preferences. The criteria values for the vector $\vec{\Psi}$ are given by the decision maker and usually meet his expectations. The vector $\vec{\Phi}$ represents the values of the criteria that the decision maker found undesirable. How to interpret and determine the preference vectors depends on which variant to calculate PVM we have chosen [12].

*Stage 8. Making the ranking (based on importance factor)*

The final value of the importance factor measure $\mu_j$ is determined by calculating the weighted average value of the factors $\underset{v}{\mu_j}$ , $\underset{d}{\mu_j}$ i $\underset{nd}{\mu_j}$ :

$$\mu_j = \frac{\underset{v}{\mu_j} m_v - \underset{d}{\mu_j} m_d + \underset{nd}{\mu_j} m_{nd}}{m_v + m_d + m_{nd}} \tag{5}$$

Negative sign at the value of $\underset{d}{\mu_j}$ factor is associated with its somewhat different character of this factor. For $\underset{v}{\mu_j}$ i $\underset{nd}{\mu_j}$ the bigger the value of the factor measure, the better decision variant, and conversely in case of $\underset{d}{\mu_j}$ , the smaller the factor's measure value, the better variant [12].

*Stage 9. Selection a decision variant (solution)*

Ranking of decision variants is made under the final value of importance factor $\mu_j$ . The bigger the value of the importance factor, the closer to decision-maker's preference the variant is.

*Stage 10. Assessment of effects of decision variant implementation*

In this stage, all objects that have no chance of achieving the first rank should be discarded and for other objects set values of subjective criteria and re-determine the importance coefficient. The procedure of discarding objects requires determining the threshold of the importance coefficient. Objects with an importance coefficient calculated for all criteria which ignore subjective criteria with a value below the threshold will not have a chance to move to the first position after taking into account subjective criteria. Due to the fact that the importance coefficient is calculated on the basis of three indicators. The threshold must be calculated on the basis of these indicators. The calculation method the threshold of the importance factor below which all objects with value below must be eliminated is presented in [38].

## 3. Results and discussion

### 3.1. Experiment 1 - selection of a country that provides the best environment for researchers

To select countries that provide the best environment for researchers, applying the VMCM method (based on selected indicators) the following ranking of countries was obtained (Table 1).

**Table 1-**Ranking of the European countries in 2017, made with the use of VMCM (for artificial and real pattern)

| ARTIFICIAL PATTERN (QUARTILE) | | | REAL PATTERN | | |
|---|---|---|---|---|---|
| Name | Measure | Class | Name | Measure | Class |
| Germany | 4,657 | 1 | Germany | 5,991 | 1 |
| Finland | 3,876 | 1 | Finland | 4,474 | 1 |
| Norway | 3,811 | 1 | Norway | 4,261 | 1 |
| Sweden | 3,606 | 1 | Austria | 4,257 | 1 |
| Denmark | 3,440 | 1 | Belgium | 4,155 | 1 |
| Luxembourg | 3,363 | 1 | Czechia | 4,127 | 1 |
| Netherlands | 3,351 | 1 | France | 4,036 | 1 |
| Austria | 3,337 | 1 | Slovenia | 4,007 | 2 |
| Belgium | 3,315 | 1 | Sweden | 4,003 | 2 |
| France | 3,090 | 2 | Denmark | 3,826 | 2 |
| Slovenia | 2,862 | 2 | Netherlands | 3,791 | 2 |
| Iceland | 2,697 | 2 | Luxembourg | 3,769 | 2 |
| Czechia | 2,668 | 2 | United Kingdom | 2,827 | 2 |
| United Kingdom | 2,598 | 2 | Iceland | 2,786 | 2 |
| Estonia | 1,872 | 2 | Hungary | 2,594 | 2 |
| Ireland | 1,813 | 2 | Slovakia | 2,519 | 3 |
| Spain | 1,528 | 3 | Estonia | 2,395 | 3 |
| Hungary | 1,382 | 3 | Spain | 2,388 | 3 |
| Slovakia | 1,242 | 3 | Italy | 2,148 | 3 |
| Lithuania | 1,232 | 3 | Lithuania | 2,022 | 3 |
| Italy | 1,023 | 3 | Ireland | 2,006 | 3 |
| Poland | 0,622 | 3 | Croatia | 1,530 | 3 |
| Croatia | 0,492 | 3 | Greece | 1,481 | 3 |
| Cyprus | 0,356 | 3 | Serbia | 1,457 | 3 |
| Latvia | 0,347 | 3 | Poland | 1,214 | 3 |
| Greece | 0,347 | 3 | Romania | 1,069 | 4 |
| Malta | 0,263 | 3 | Latvia | 0,956 | 4 |
| Serbia | 0,109 | 4 | Bulgaria | 0,941 | 4 |
| Portugal | 0,029 | 4 | Portugal | 0,935 | 4 |
| Bulgaria | -0,224 | 4 | Turkey | 0,767 | 4 |
| Romania | -0,238 | 4 | Malta | 0,712 | 4 |
| Turkey | -0,311 | 4 | Cyprus | 0,646 | 4 |
| Macedonia | -0,945 | 4 | Macedonia | 0,000 | 4 |

In the first case, classification was made referring to the artificial pattern, as the difference between a pattern vector (object) and a vector being anti-pattern. Coordinates of the pattern for stimulants is the value of III quadrille, and for destimulants the value of I quadrile. The coordinates of anti- pattern for stimulants is the value of I quartile, for destimulants is the value of III quartile. The created ranking shows that in total 9 counties were classified to Class 1: Germany, Finland, Norway, Sweden, Denmark, Luxembourg, Netherlands, Austria, Belgium. These are countries providing potentially the best environment for researchers. Germany is the leading country in this ranking. The opposite pole (Class 4) includes also 6 countries, of which Macedonia had the lowest ranking.

Next, it was checked, which of the countries under analysis was close to Germany, which was ranked the best. The pattern vector, which is the reference object, in this case is the difference between the vector representing Germany (the highest place in the ranking) and the vector representing Macedonia (the lowest place). In this way, the real pattern vector was constructed. As you can see (Table-1) there are still 6 countries in Class 1 (outside Germany). In this respect, the Czech Republic changed their ranking from place 13 to 6, and France from place 10 to 7. Both countries moved up from Class 2 to 1. This shows that in the context of friendliness for researchers these countries are closer to Germany. In turn, Sweden, Denmark, Luxembourg, Netherlands classified in Class 1 moved to Class 2. You can also see changes at the end of ranking. Cyprus was the next to last, which at the same time changed its position from Class 3 to 4. Likewise Latvia and Malta moved down from Class 3 to 4. While Serbia moved up from Class 4 to 3. The graphical representation of the countries' distribution into classes is presented in Figure-1.

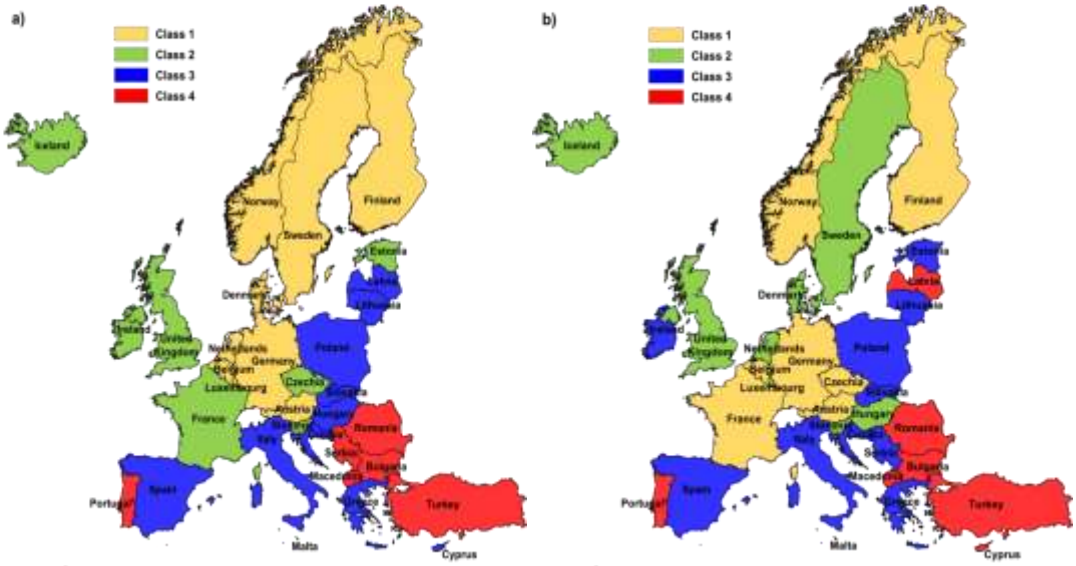


**Figure 1-**Classification of countries (VMCM) according to friendliness for researchers a) artificial pattern, b) real pattern.

Using the VMCM method as a reference vector, depending on the considered scenario, you can accept any object (state). This may be, for example, a non-European country or one of the analyzed countries, e.g. Romania. The reference vector does not necessarily have to be the best object of the sample in question. The VMCM method eliminates from other methods, e.g. TOPSIS. The measure is not limited either from the bottom or from the top, thus accepts objects better than the pattern, allows adding objects from outside the sample without the need to create a new pattern and is more sensitive to the dynamics of change (enables the study of dynamics). A detailed description of this situation can be found in Piwowarski et al. (2018).

### 3.2. Experiment 2 - selection of the most friendly country for seniors

In this study, the PVM method is used, due to the possibility of taking into account a greater number of criteria and decision alternatives (in relation to other multiple criteria methods). Usually, in this type of method, the ranking is usually created based on the pairwise comparison. With a larger number of assessment criteria and considered decision-making alternatives, a large number of comparisons have to be made. For example, in the AHP method [29,39, 14] the number of comparisons between the decision variant (*n*) is calculated based on the formula:

$$\frac{n^2-n}{2} \quad (6)$$

For the conducted studies on selection of the most friendly country for seniors (4 criteria, 30 alternatives), 435 comparisons have to be made for each criterion. For all criteria, it would be 1740 comparisons. This is a very large number that practically eliminates this method from such applications. Even with a much smaller number of evaluation criteria and considered alternatives, the pairwise comparison process becomes ineffective. After some time, the decision maker, after some number of comparisons made, becomes tired (cognitive fatigue). These issues (the effectiveness of the pairwise comparison procedure), and thus the applicability of such methods were the subject of the research [40]. The subject of the research was the selection of a car (6 alternatives) based on 6 criteria. The results of the studies showed that after 15 comparisons made by the participants of the experiment, the efficiency dropped down and fatigue appeared (Figure-2).

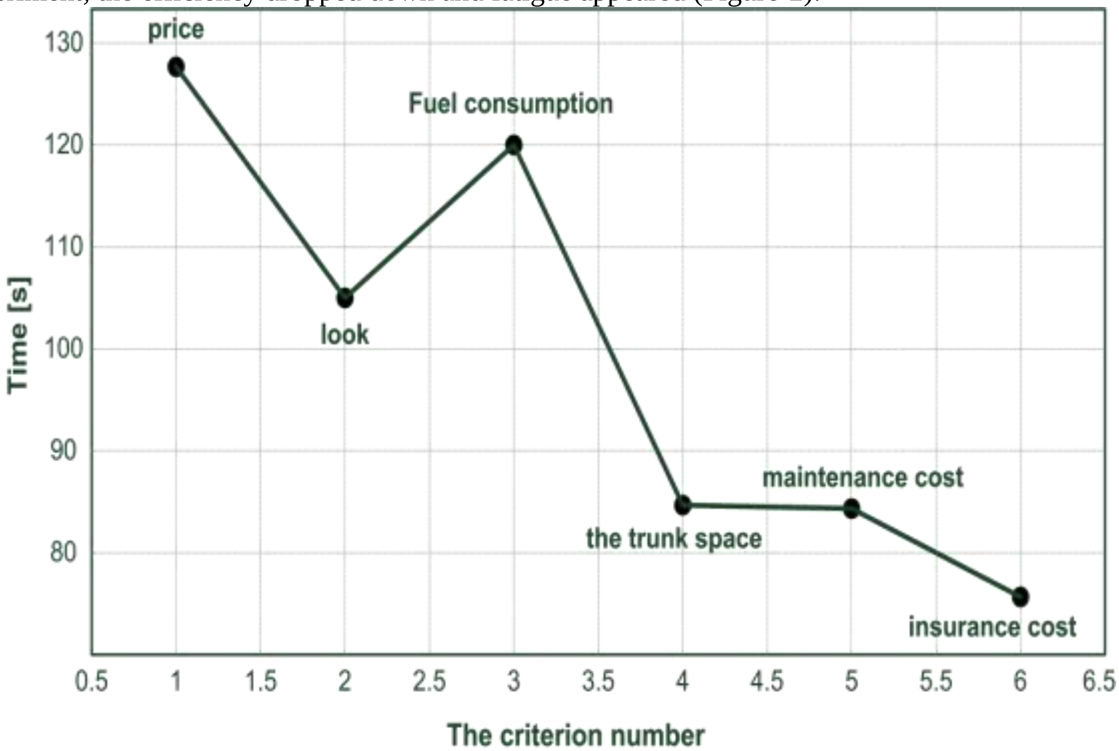


**Figure 2-**Data from the web-tracking system: time of pairwise comparisons of alternatives relative to individual criteria.

Initially, for one of criteria 'price' , the time for comparing alternatives was over 2 minutes. Later, for the 'appearance' criterion, it dropped down and for the 'cost-efficiency' criterion rose. For the criterion 'trunk capacity' dropped down to 1.5 minute and for the other criteria was more or less the same. It could have resulted from better skills in completing a questionnaire by a questioned person and a decrease in interest in the survey, which may have led to the selection of random answers.

The solution to this problem (fatigue when making decisions) is to apply a method in which a decision maker will not have to make such a large number of comparisons. The method, which includes participation of a decision-maker, but at the same time minimizes his or her involvement, is the PVM method. In this method, the participation of a decision maker is necessary in the initial phase of a decision-making process (determination of the value of a preference vector) and in situations in which quality criteria are considered [41, 42].

In the first stage of the study, European countries were classified according to 4 quantitative criteria related to the assessment of senior – friendliness (At-risk-of-poverty rate of older people; Median relative income of elderly people (60+); Individuals using the internet for interaction with public authorities; 4.Social benefits other than social transfers in kind, payable, Percentage of gross domestic product (GDP). As a result of the ranking, countries were divided into four classes. Countries classified to the Class 1 were characterized by the most desirable results based on the preferences considered (Figure-3):

1. Finland,
2. Denmark,
3. Norway,
4. France.

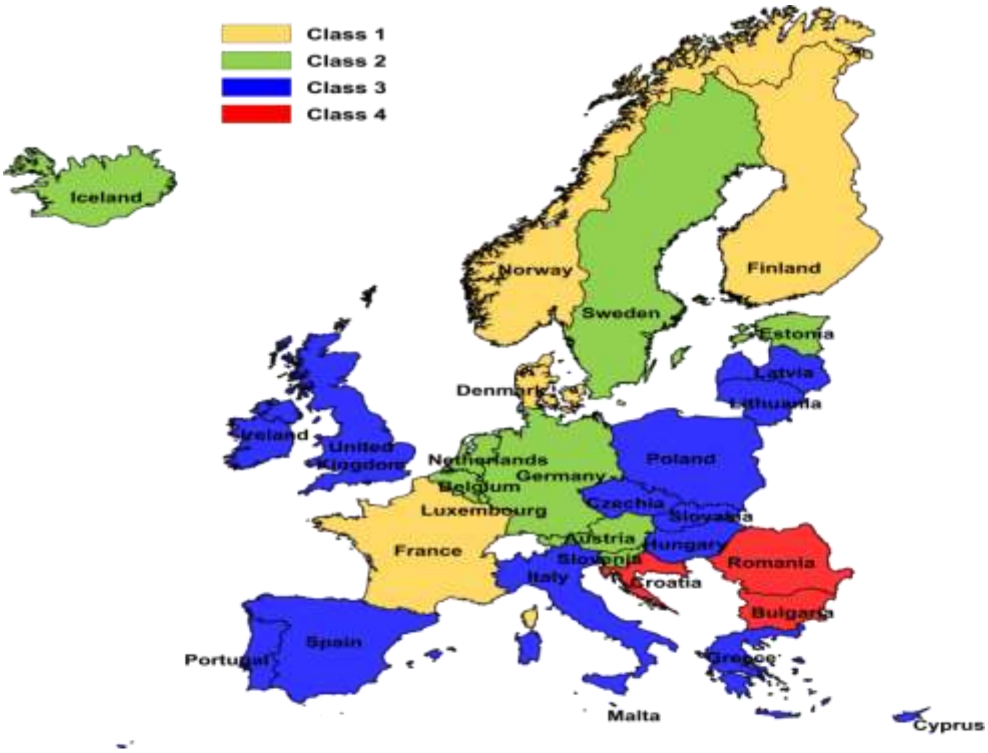


**Figure 3-**Ranking (classification) of European counties according to senior-friendly living conditions

Then, an additional criterion 'climate' was introduced, which may be important for seniors. This is a quality criterion. The repeated calculation procedure aimed at obtaining the final ranking of countries included 4 decision alternatives, according to the procedure of eliminating decision alternatives in PVM method. Obtained alternatives (countries) coincide with those, which were chosen in the first selection. Evaluation criteria included earlier criteria and in addition 'climate' [43,44].

The example of pairwise comparison of decision alternatives according to 'climate' criterion is presented below (7):

$$\begin{array}{c} \\ FI \\ DK \\ NO \\ FR \end{array} \begin{array}{c} \begin{array}{cccc} FI & DK & NO & FR \end{array} \\ \begin{bmatrix} 1 & 1/7 & 1/2 & 1/8 \\ 7 & 1 & 5 & 1/3 \\ 2 & 1/5 & 1 & 1/5 \\ 8 & 3 & 5 & 1 \end{bmatrix} \end{array} \tag{7}$$

Obtained weights of decision alternatives according to 'climate' index are as follows: Finland – 0.21; Denmark – 1.31; Norway – 0.37; France – 2.36. Obtained values (own vector) for a 'climate' criterion were filled in a table including values standardized for the other criteria (Table-2).

**Table 2-**Standardized data to choose a country for living considering an additional 'climate' criterion

| Object (country) | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|---|---|---|---|---|---|
| Finland | 2,10 | 2,35 | 2,55 | 2,72 | 0,21 |
| Denmark | 2,51 | 2,16 | 2,73 | 2,3 | 1,31 |
| Norway | 2,54 | 2,59 | 2,58 | 2,10 | 0,37 |
| France | 2,78 | 2,84 | 2,08 | 2,80 | 2,36 |

The results obtained through PVM (rank of countries) according to the additional criterion 'climate', are presented in Table -3.

**Table 3**-Result of the ranking of European countries according to the additional criterion 'climate'

| Object (country) | Ranking |
|---|---|
| France | 3,949 |
| Denmark | 2,081 |
| Norway | 0,461 |
| Finland | 0,315 |

Ranking of senior-friendly countries, including additional criterion 'climate' represents that France is the best and Finland is the worst choice. Comparing to the original ranking (without the 'climate' criterion) it is visible, that this criterion was so important to a decision-maker that ranking was totally changed.

**4. Conclusions**

Carried out studies showed that application of the proposed methods (VMCM, PVM) in solving both macro (limited participation of a decision-maker) and micro (significant participation of a decision-maker) problems are relevant. VMCM method that applies vector calculus properties to construct an aggregated vector measure, delete limitations of the other methods, e.g. HELLWIGA. This refers to e.g. opportunity to include objects which are better than a defined pattern. In addition, it allows adding objects from outside of the sample without a pattern conversion, in addition it are more sensitive to dynamics of changes occurring. This property of VMCM method makes this method perfectly suitable to study economic phenomena, which require change in time analysis. The presented study of a country which creates the best environment for researchers is the perfect example. Likewise, VMCM method can be applied to study other aspects of socio - economic development. PVM is applied where other multi-criteria methods (e.g. AHP) do not work. This involves a large number of pairwise comparison and related fatigue issue. Due to a relatively simple application it may be used as an alternative to methods so far discussed in literature. The advantage of PVM method is that active participation of a decision-maker in a decision process is not required. Before, a decision maker may specify and express his preferences in a form of two vectors: motivating and demotivating one. Such approach allows solving decision problems with practically unlimited number of criteria and decision alternatives, what is impossible in case of AHP, ANP or some versions of ELECTRE methods. PVM method is also relatively simple to implement in programming languages supporting matrix operations, so a process supporting a decision making can be relatively easy automated. This method is relevant in making rankings or where one decision alternative out of many other is to be selected.

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