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A New Structure of Random Approach Normed Space via Banach Space

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Abstract

The goal of this research is to define the convergent sequence in A-random approach space and sequentially convergent are discussed and the cluster point, open and closed ball and linear transformation. We are going to explain a new structure of Random approach normed space via Banach space in and discussed all the relations between metric space in this research.

Keywords: Approach space, Random space, Approach normed space, Banach Approach space.

بنية جديدة للفضاء المعياري التقاربي العشوائي بوسطة فضاء باناخ

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قسم الرياضيات , كلية التربية , جامعة القادسية , القادسية , العراق

الخلاصة

الهدف من هذا البحث هو تحديد التسلسل المتقارب في فضاء الاقتراب العشوائي و مناقشة التقارب المتسلسل ونقطة التجمع والكرة المفتوحة والمغلقة والتحول الخطي. سنشرح بنية جديدة للنهج العشوائي للفضاء المعياري عبر فضاء باناخ وناقشنا في هذا البحث جميع العلاقات مع الفضاء المتري.

1. Introduction

A. N. Serstnev in [1] who was illustrated Random (probabilistic) normed spaces via means of a definition that was closely modeled on the theory of normed spaces which is classical, A. N. Serstnev employed to study the issue of preferable approximation in statistics. In the sequel, we shall take on usual terminology, notation, and conventions of the theory of random normed spaces, as in [2], [3], [4]. The distance between points and sets in a metric space were studied by sue R. Lowen in [5].In topological space one analogously has that the distance between points and sets are given by the closure operator. The measures of Lindelof and separability in approach spaces were studied via R. Baekeland and B. Lowen in [6].The development of the fundamental theory of approximation was studied R. Lowen in [7].There are two types of Cauchy structures, approach Cauchy structure and ultra-approach Cauchy structure, according to R. Lowen and Y. Jin Lee in [8]. R. Lowen and M. Sioen introduced the definitions of separation axioms in approach spaces and determined their relation to each other in and [9], [10]. An approach groups spaces, semigroup spaces, and uniformly convergent are acquainted via R. Lowen and B. Windels in [11]. In [12], R. Lowen, M. Sioen and D. Vaughan acquainted a complete theory for all approach spaces with an underlying topology that agrees with the usual metric completion theory for metric spaces. Approach

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vector spaces are studied via R. Lowen and S. Verwuwlgen,in [13]. The relationship between Functional ideas and Topological theories are found via R. Lowen, C. Van Olmen and T. Vroegrijk in [14]. In [15], G. C. Brümmer and M. Sion sophisticated abicompletion theory for the category of approach spaces in sense of Lowen [16] which extends the completion theory obtained in [11]. In [17], A. Roldán , J. Martínez-Moreno and C. Roldán acquainted the notion of Fuzzy approach spaces generalization of Fuzzy metric spaces and proved some properties of Fuzzy approach spaces. R. Lowen and C. Van Olmen [18] discussed some notions and relations in approach theory. The notion of cocompleteness for approach spaces and proved some properties in cocompleteness approach space were studied via G. Gutierres and D. Hofmann in [19] . A new isomorphic characterizations of approach spaces, pre-approach spaces were given K. Van Opdenbosch in [20], convergence approach spaces, uniform gauge spaces, topological spaces and convergence spaces, topological spaces, metric spaces, and uniform spaces. In R. Lowen and S. Sagiroglu [21]. And in B. Y. Hussein and R. K. Abbas [22] through which you can find out Normed approach space , so Banach approach space. In B. Y. Hussein and S. Saeed [23] defined the distance between two different sets in approach normed space , topological approach Banach space . In this paper the concepts of random and approach were combined by a relationship explained in the research throughout which the concepts random approach vector space, random approach normed space, random approach Banach space and based.

This paper is divided into five sections: Section one introduces the introduction of the research. In section two, new results in convergent sequences in a-random approach spaces are proved. We also explain the relationship complete and complete in a-random approach space. In section three, we introduce the definition of a-random approach normed space and prove some results in a-random approach normed space. In section four, a new result in δ_R-contractions on a-random approach normed spaces.

2. Convergent results in a-random approach space.

In this section, we define the convergent of sequence in a-random approach (or shortly, appr.) space by the following definitions:

Definition 2.1: Let (Ω, d_R) be a metric space, then a sequence $\{ a_n\}_{n=1}^{\infty}$ in Ω is said to be a right Cauchy sequence if for all $\varepsilon > 0$ there exists $k \in Z^+$ such that
$d_R(a_m , b_n) < \varepsilon$ for $m,n \leq k , m \geq n$. Left Cauchy sequence $if\ for\ all\ \varepsilon > 0$ there exists $k \in Z^+$such that $d_R(a_n , b_m) < \varepsilon$ for $all\ m,n \leq k , m \geq n$. If a sequence is left and right Cauchy is called Cauchy sequence.

Definition 2.2: A set $N \in 2^{\Omega}$is said to be a cluster point in an a-random appr. space (Ω, δ_R) if there exists disjoint sequence $\{ a_n\}_{n=1}^{\infty}$ in Ω such that $\inf_{x\in N} \delta_R(\{ a_n \}, N) = K_0(0)$, which is written by $\{ a_n\}_{n=1}^{\infty} \rightarrow N$. We denoted the set of all cluster point in A-random appr. space $\Psi(\Omega)$.

Definition 2.3: A sequence $\{ a_n\}_{n=1}^{\infty}$ in Ω is said to be Cauchy sequence in A-random appr. space $\delta_R -$ Cauchy if for every cluster point N , $\lim_{n\to\infty} \inf_{x\in N} \delta_R(\{ a_n \}, N) = K_0(0)$ sequence $\{ a_n\}_{n=1}^{\infty}$ in Ω is said to be δ_R- convergent sequence in A-random appr. space if there exist $x \in \Omega$ for all
$N \in \Psi(\Omega)$, $\delta_R(\{a_n \}, N) = K_0(0)$

Proposition 2.4: Let(Ω, δ_R) be a-random appr. space, then the following are equivalent:

1) $\{ a_n\}_{n=1}^{\infty}$ be disjoint δ_R- Convergent sequence in a-random appr. space.

2) $\lim_{n\to\infty} \inf_{x\in N} \delta_R(\{a_n\}, N) = K_0(0)$ and $\lim_{n\to\infty} \sup_{x\in N} \delta_R(\{a_n\}, N) = K_0(0)$

Proof: *Let* $\{a_n\}_{n=1}^{\infty}$ be disjoint δ_R- convergent sequence in A-random appr. space. Then there exist $x \in \Omega$ for *all* $N \in \Psi(\Omega)$: $\delta_R(\{a_n\}, N) = K_0(0)$

For all $N \in \Psi(\Omega)$: $\lim_{n\to\infty} \inf_{x\in N} \delta_R(\{a_n\}, N) = K_0(0)$ and $\sup_{x\in M} \delta_R(\{a_n\}, M) = K_0(0)$

For all $N \in \Psi(\Omega)$: $\lim_{n\to\infty} \inf_{x\in N} \delta_R(\{a_n\}, N) = K_0(0)$

And $\lim_{n\to\infty} \sup_{x\in N} \delta_R(\{a_n\}, N) = K_0(0)$

Conversely, suppose the condition (2) is true. $\lim_{n\to\infty} \inf_{x\in N} \delta_R(\{a_n\}, N) = K_0(0)$ and $\lim_{n\to\infty} \sup_{x\in N} \delta_R(\{a_n\}, N) = K_0(0)$.

Then N is cluster point, that is . $\inf_{x\in N} \delta_R(\{a_n\}, N) = K_0(0)$

Then there exists $x \in \Omega$ for all $N \in \Psi(\Omega)$: $\delta_R(\{a_n\}, N) = K_0(0)$

Thus $\{a_n\}_{n=1}^{\infty}$ be δ_R- convergent sequence in a-random appr. space.

Remark 2.5: Every δ_R- convergent sequence is $\delta_R -$ Cauchy(Cauchy A-random appr.space).

Proposition 2.6: If (Ω, δ_R) is a-random appr. space then following are equivalent:

1) $\{a_n\}_{n=1}^{\infty}$ is δ_R- convergent sequence in A-random appr. space;

2) $\sup_{N \in \Psi(\Omega)} \inf_{x\in N} \delta_R(\{a_n\}, \{x\}) = K_0(0)$.

Proof: Suppose that $\{a_n\}_{n=1}^{\infty}$ is disjoint δ_R- convergent sequence in a-random appr. space. There exist $x \in \Omega$ for all

$N \in \Psi(\Omega)$: $\delta_R(\{a_n\}, N) = K_0(0)$.

And $\inf_{x\in N} \delta_R(\{a_n\}, N) = K_0(0)$, then $\lim_{n\to\infty} \inf_{x\in N} \delta_R(\{a_n\}, N) = K_0(0)$.

And $\sup_{x\in N} \delta_R(\{a_n\}, N) = K_0(0)$ *that is* $\lim_{n\to\infty} \sup_{x\in N} \delta_R(\{a_n\}, N) = K_0(0)$.

Then, $\sup_{N \in \Psi(X)} \inf_{x\in N} \delta_R(\{a_n\}, \{x\}) = K_0(0)$.

Conversely, it is clear.

Proposition 2.7: If (Ω, δ_R) is a-random appr. metric space and $\{a_n\}_{n=1}^{\infty}$ be disjoint sequence in Ω, then it is Cauchy sequence in (Ω, δ_R) if and only if is δ_R- Cauchy sequence in (Ω, δ_R) .

Proof:

Let $\{a_n\}_{n=1}^{\infty}$ be Cauchy sequence in (Ω, δ_R) , then we have that $\inf_{x\in N} \delta_R(\{a_n\}, N) = K_0(0)$

$\inf_{x\in N} \delta_R(\{a_n\}, \{a_m\}) = \inf_{x\in N} \delta_R(\{a_n\}, \{a_m\}) = K_0(0)$.

That is $\delta_R(\{a_n\}, \{a_m\}) = K_0(0)$.

Then $\{a_n\}_{n=1}^{\infty}$ is left Cauchy sequence.

That is $\delta_R(\{a_m\}, \{a_n\}) = K_0(0)$. Then $\{a_n\}_{n=1}^{\infty}$ is right Cauchy sequence.

Thus, $\{a_n\}_{n=1}^{\infty}$ is Cauchy sequence in (Ω, δ_R) .

Conversely, if $\{a_n\}_{n=1}^{\infty}$ is a Cauchy sequence in (Ω, δ_R) .

Then it is left and right Cauchy sequence, for all $\varepsilon < 0$, hence there exists $k \in Z^+$ such that $\delta_R(\{a_m\}, \{a_n\}) < \varepsilon$, for all $m, n \leq N$, $m \geq n$ and for all $\varepsilon < 0$ there exists $k \in Z^+$ such that $\delta_R(\{a_n\}, \{a_m\}) < \varepsilon$, for all $m, n \leq N$, $n \geq m$.

Hence $\{a_n\}_{n=1}^{\infty}$ is δ_R- Cauchy sequence in a-random appr. space.

Theorem 2.8: Let (Ω, δ_R) be an a-random appr. space, $\langle a_n \rangle$ and $\langle b_n \rangle$ be δ_R – converge Sequence in (Ω, δ_R) to a , b Respectively, then:

1) $\langle a_n + b_n \rangle$is an δ_R – converge to $a + b$.

2) $\langle \omega a_n \rangle$ is an δ_R – converge to ωa.
3) $\langle a_n . b_n \rangle$ is an δ_R – converge to $a.b$.
The proof is clear .

Theorem 2.9: A-random appr. topological space is a topological space (Ω, T_R) that associated with natural a- Random appr. space, we define a function
$\delta_R: \Omega \times 2^{\Omega} \to \nabla^+$ by:

$$\delta_{T_R}(x, B) = \begin{cases} K_0(0) & if \ x \in CL(B) \\ K_0(\infty) & if \ x \notin CL(B) \end{cases}$$

for all $x \in \Omega$, $, B \in 2^{\Omega}$, $(\Omega, \mathrm{T}_R, \delta_{T_R})$ for topology T_R on Ω is called a topological a-random appr. space, and δ_{T_R} is called topological δ_{T_R}-distance.

Definition 2.10 : Let (Ω, δ_R) be a-random appr. space. For $x \in \Omega$ the center at x and of radius $r > 0$ is the set $\mathrm{H}_r(x) = \{ s \in \Omega, \delta_R(s, \{x\}) > r \}$, where the set H_r is called δ_R – open ball.

Definition (2.11): Let Ω A-random appr. vector space on field F.A topological A-random appr. vector space Ω with an induced topology T_Ω satisfy two axioms:
1) The map $+ : \Omega \times \Omega \to \Omega, (a, b) \to a + b$ is δ_R –contraction .
2) The map: $F \times \Omega \to \Omega \ is \ \delta_R$ –contraction.
When it is written as (Ω, T_Ω).

Proposition 2.12: Let (Ω, T_Ω) be a topological space, then the function
$\delta_R: \Omega \times 2^{\Omega} \to \nabla^+$

defined by : $$\delta_{T_R}(x, B) = \begin{cases} K_0(0) & if \ x \in CL(B) \\ K_0(\infty) & if \ x \notin CL(B) \end{cases}$$

is δ_R -distance on Ω.

Proof: We prove that δ_R is indeed a distance
1) Since $x \in CL(B) \quad then \ \delta_R(x, B) = K_0(0)$
2) $we \ know \ that \quad CL(\emptyset) = \emptyset, \ then \ \delta_R(\emptyset, B) = K_0(\infty)$
3) $For \ all \ H, B \in 2^{\Omega}, since \ CL(x, B \cup H) = CL(B) \cup CL(H) =$
$min\{\delta_R(x, B), \delta_R(x, H)\} = min\{CL(B), CL(H)\} = min\{CL(B), CL(H)\} =$
$min\{\delta_R(X, B), \delta_R(X, H)\}$.
4) $For \ all \ B \in 2^{\Omega}$and for all $g(t) \in \nabla^+$ $, We \ have \ B^{g(t)} = CL(B) \ and \ B^{K_0(\infty)} =$ Ω
$this \ gives \ us \ \delta_R(X, B) \geq \delta_R\left(X, B^{g(t)}\right) + g(t)$
Hence $\delta_{T_R}(x, B) \ is \ \delta_R$ – distance on Ω.

Theorem (2.13) : Let $(\Omega, \delta_{R_\Omega})$ be a-random appr. vector space , B be Closed a-random appr. sub space of Ω . Then $(\Omega/B, \delta_{R_{\Omega/B}})$ is a-random appr. vector space , and we define
$\delta_{R_{\Omega/B}} : \Omega/B \times 2^{\Omega/B} \to \nabla^+$ as follows : $\delta_{R_{\Omega/B}}(x, U) = \acute{\delta}_R(x + B, U + B) = \delta_R(x, U)$

Proof: We will prove $\acute{\delta}_R$ satisfy distance condition:

1) $\acute{\delta}_R\ (x\ +\ B\ , U + B\ ) = \delta_R(x, U)$

2) $If\ \ U = \emptyset\ , \acute{\delta}_R\ (x\ +\ B\ , \emptyset) = \delta_R\ (\ x\ , \emptyset) = K_0(\infty)\ \ , If\ \ U \neq\ \emptyset\ then\ \ \delta_R\ (x\ , U) = K_0(0)\ , x\ \in\ U$

$$\acute{\delta}_R\ (x\ +\ B\ , U + B\ )\ = \acute{\delta}_R(x\ , \{x + B\})\ =\ \delta_R\ (x, U)\ = K_0(0)$$

3) $\acute{\delta}_R\ (x\ +\ B\ , U + B \cup N + B\ )\ = \delta_R\ (x\ , U \cup N)$

$$=\ min\ \{\ \delta_R(x, U)\ , \delta_R(x, N)\ \}$$

4) $\acute{\delta}_R(\ x\ + B\ , U + M\ )\ \ =\ \ \delta_R\ (x\ , U\ )\ \ \ \geq\ \ \delta_R\ (\ x\ , U^{g(t)}) \ + g(t)$

$$= \delta_R\ (x\ +\ B, U^{g(t)}\ +\ B)\ +\ g(t)$$

Definition 2.14 : Let (Ω, δ_R) be a-random appr. space a sequence $\{a_n\}$ is convergent sequence in the a-random appr. space to $N\ \subseteq\ \Omega\ \ if\ \ \lim\limits_{n\to\infty}\ \inf\limits_{a\epsilon N}\ \delta_R\ (\{a_n\}, N) = K_0(0)$ and $\lim\limits_{n\to\infty}\ \ \sup\limits_{a\epsilon N}\ \delta_R\ (\{a_n\}, N) = K_0(0)$.

Definition 2.15 : Let (Ω, δ_R) and $(\mathrm{E}, \acute{\delta}_R)$ are a-random appr . spaces. The function $\xi\ :\ \Omega \to E$ is called sequentially contraction if $\lim\limits_{n\to\infty}\ \delta_R\ (\{\xi(a_n)\}, \xi(N)) = K_0(0)$ Whenever $\lim\limits_{n\to\infty}\ \ \delta_R\ (\{a_n\}, N) = K_0(0)$.

Definition 2.16 : Let Ω and E be two a-random appr. vector spaces on A-random appr over the same field F, a mapping: $\Gamma\colon \Omega \to E$ is said to a-random appr. linear transformation if the following hold :

1) $\Gamma(a + b) =\ \Gamma(a) *\ \Gamma(b)$.

2) $\Gamma\ (\ \lambda a)\ =\ \lambda\Gamma(a)\ \ \ for\ all\ \ \lambda\ \in\ \mathrm{F}\ ,\ \ for\ all\ a, b\ \in\ \Omega$.

Definition 2.17 : Let $\Gamma\colon \Omega \to E$ be a a-random appr. linear transformation. Then the set $\delta_R - ker\ (\Gamma)\ = \{B \subseteq \Omega : \Gamma(B) = \{0\}\ \ \}\ =\ \Gamma^{-1}(\{0\}\ )$ is called the a-random appr. kernel of Γ.

Theorem 2.18 : Let $(\Omega, T_\Omega, \delta_R)$ and $(E, T_{\mathrm{E}}, \acute{\delta}_R)$be a topological a-random appr. vector spaces. And the approach linear map $\Gamma\colon \Omega \to E$ is contraction then that is $ker(\Gamma)$ is closed .

Proof: Suppose Γ is the δ_R-contraction.

To prove $Ker(\Gamma)$ is closed set, let $\{a_n\}$ be a disjoint sequence that convergent to $a\ in\ Ker(\Gamma)$ such that $\lim\limits_{n\to\infty}\ \ \inf\limits_{a\epsilon \ker(\Gamma)}\ \ \delta_R\ (\{a_n\}, N) = K_0(0)$ and $\lim\limits_{n\to\infty}\ \ \sup\limits_{a\epsilon \ker(\Gamma)}\ \ \delta_R\ (\{a_n\}, N) = K_0(0)$

Since Γ is δ_R- contraction, that is $\acute{\delta}_R(\ \Gamma(\{a_n\}), \Gamma(N\ )) \geq\ \ \delta_R(\{a_n\}, N\ )$.

Then,

$$K_0(0) = \lim_{n\to\infty} \inf_{x\epsilon N}\ \ \delta_R\ (\{a_n\}, N)\ \geq\ \lim_{n\to\infty} \inf_{x\epsilon N}\ \ \ \acute{\delta}_R(\ \Gamma(\{a_n\}), \Gamma(N))\ \geq$$

$$\lim_{n\to\infty} \sup_{x\epsilon N}\ \ \ \acute{\delta}_R(\ \Gamma(\{a_n\}), \Gamma(N))\ \ \geq\ \ \ \lim_{n\to\infty} \sup_{x\epsilon N}\ \ \delta_R(\{a_n\}, N) = K_0(0)\ .$$

$$\lim_{n\to\infty} \sup_{x\epsilon N}\ \ \ \acute{\delta}_R\big(\ \Gamma(\{a_n\}), \Gamma(N)\big) = K_0(0)\ and\ \lim_{n\to\infty} \inf_{x\epsilon N}\ \ \ \acute{\delta}_R\big(\ \Gamma(\{a_n\}), \Gamma(N)\big) = K_0(0).$$

$$(\ \Gamma(\{a_n\})\ = K_0(0), \lim_{n\to\infty}\acute{\delta}_R\big(\ \Gamma(\{a_n\}), \Gamma(N)\big) = K_0(0)\ \ then\ \Gamma(\{a\})\ = K_0(0)\ , a\ \in\ Ker\ (\Gamma).$$

Now, suppose $Ker\ (\Gamma)$ is closed set, let $\{a_n\}$ be disjoint sequence convergent to A in δ_R -$Ker\ (\Gamma)$, to prove $\Gamma(\{a_n\})$ convergent to $\Gamma(\{a\})$, since $\delta_R\ - Ker\ (\Gamma)$ is closed, $x \in \delta_R\ - Ker\ (\Gamma)$, assume that $\Gamma(\{a_n\})$ is not convergent to $\Gamma(\{0\})$ in N, that is $\Gamma\ is$ not $\delta_R\ -$contraction

Then $\lim_{n\to\infty} \sup_{x\epsilon N} \acute{\delta_R}(\Gamma(\{a_n\}),\Gamma(N)) \neq K_0(0)$ or $\lim_{n\to\infty} \inf_{x\epsilon N} \acute{\delta_R}(\Gamma(\{a_n\}),\Gamma(N)) \neq K_0(0)$.
If $\lim_{n\to\infty} \sup_{x\epsilon N} \acute{\delta_R}(\Gamma(\{a_n\}),\Gamma(N)) \neq K_0(0)$ or $\lim_{n\to\infty} \inf_{x\epsilon N} \acute{\delta_R}(\Gamma(\{a_n\}),\Gamma(N)) = K_0(0)$
$\lim_{n\to\infty} \inf_{x\epsilon N} \delta_R(\{a_n\},N) > \lim_{n\to\infty} \inf_{x\epsilon N} \acute{\delta_R}(\Gamma(\{a_n\}),\Gamma(N))$
then $\acute{\delta_R}(\Gamma(\{a_n\}),\Gamma(N)) < K_0(0)$,this impossible
If $\lim_{n\to\infty} \sup_{x\epsilon N} \acute{\delta_R}\big(\Gamma(\{a_n\}),\Gamma(N)\big) = K_0(0)$ *or* $\lim_{n\to\infty} \inf_{x\epsilon N} \acute{\delta_R}(\Gamma(\{a_n\}),\Gamma(N)) \neq K_0(0)$
$0 = \lim_{n\to\infty} \sup_{x\epsilon N} \delta_R(\{a_n\},N) \geq \lim_{n\to\infty} \sup_{x\epsilon N} \acute{\delta_R}\big(\Gamma(\{a_n\}),\Gamma(N)\big) = K_0(0)$ which is impossible.
If $\lim_{n\to\infty} \sup_{x\epsilon N} \acute{\delta_R}(\Gamma(\{a_n\}),\Gamma(N)) \neq K_0(0)$ and $\lim_{n\to\infty} \inf_{x\epsilon N} \acute{\delta_R}(\Gamma(\{a_n\}),\Gamma(N)) \neq K_0(0)$But,
$\Gamma(\{a_n\}) \in \delta_R - Ker(\Gamma)\ then\ \lim_{n\to\infty} \sup_{x\epsilon N} \acute{\delta_R}(0,\Gamma(N)) \neq K_0(0)\ and$
$\lim_{n\to\infty} \inf_{x\epsilon N} \acute{\delta_R}\big(0,\Gamma(x)\big)$,that is $\acute{\delta_R}\big(0,\Gamma(x)\big) \neq 0$, $\Gamma(x) \neq K_0(0)$
So, $a \notin \delta_R - Ker$ (£) this impossible. Hence Γ sequentially contraction, then Γ is δ_R −contraction.

3. Structure of a-random approach normed space

Definition 3.1: A triple (Ω,σ,T) is said to be a-random normed space, where E be a non-empty vector space, ψ is continuous t-norm and σ is mapping from E into ∇^+ such that the following condition hold.

1. $AR1)\ \boldsymbol{\sigma}_x(r) = K_0(r)\ if\ and\ only\ if\ x = 0\ , for\ any\ r > 0.$
2. $AR2)\ \boldsymbol{\sigma}_{\lambda x}(r) = \boldsymbol{\sigma}_x\left(\frac{r}{|\lambda|}\right)\ ,\ where\ \ , for\ all\ x \in \Omega.$
3. $AR3)\ \lim_{\lambda\to 0} \boldsymbol{\sigma}_{\lambda x}(r) = K_0(r)$.
4. $AR4) \boldsymbol{\sigma}_{x+y}(r+e) \geq \psi\left(\boldsymbol{\sigma}_x(r),\boldsymbol{\sigma}_y(e)\right)\ ,\ for\ any\ x,y \in \Omega\ ,\ r,e \geq 0.$
5. $AR5)\ \delta_R(r,B) = \sup_{x\in\Omega} \inf_{a\in B} \boldsymbol{\sigma}_{x-a}(r).$

Proposition 3.2 : Every a-random appr. normed space is a-random normed space.

Remark 3.3:A-random normed space is not necessary a-random appr. normed .

Definition 3.4: A-random appr. Banach space is δ_R −complete a-random appr. normed space .

Proposition 3.5 : Let Ω be finite δ_R-dimensional A-random appr. normed space is δ_R-complete and consequent A-random appr. Banach space.

Proof: : Assume $\dim(\Omega) = n > 0$, $\{\eta_1,\eta_2,\ldots,\eta_n\}$ is basis of Ω, Ω is finite δ_R-dimensional A-random appr. normed space
Let$\{a_m\}_{m=1}^{n}\ be\ a\ \delta_R$ −Cauchy sequence in Ω , $\lim_{n\to\infty} inf_{x_m\in A}\ \delta_R(\{x_m\},A) = K_0(0)$. $for\ x_m = \Sigma_{i=1}^{n} \alpha_{jm}\varphi_j$, $y_i = \Sigma_{i=1}^{n}\alpha_{ii}\eta_j$
$K_0(0) = \lim_{n\to\infty} inf_{\Sigma_{i=1}^{n}\alpha_{jm}\eta_j\in A}\ \delta_R(\Sigma_{i=1}^{n}\alpha_{im}\eta_j,A)$
$= \lim_{n\to\infty} inf_{\Sigma_{i=1}^{n}\alpha_{jm}\eta_j} inf_{y\in A}\ d(\Sigma_{i=1}^{n} a_{im}\eta_j,A)$
$= \lim_{n\to\infty} inf_{\Sigma_{i=1}^{n}\alpha_{jm}\eta_j} inf_{y\in A}\ d_{\delta_R\|\cdot\|}(\Sigma_{i=1}^{n}\alpha_{im}\eta_j,y)$

$= \lim_{n\to\infty} \; inf_{\Sigma_{i=1}^{n} \alpha_{jm}\,\eta_j} inf_{y\in A} \; d_{\delta_R\|.\|}(\Sigma_{i=1}^{n} \alpha_{im}\eta_j \,, \Sigma_{i=1}^{n} \alpha_{ii}\varphi_i \;)$
$= \lim_{n\to\infty} \; inf_{\Sigma_{i=1}^{n} \alpha_{jm}\,\eta_j} inf_{y\in A} \; \|\Sigma_{i=1}^{n} \alpha_{im}\eta_j \,, \Sigma_{i=1}^{n} \alpha_{ii}\eta_j\|$; that is$\Sigma_{i=1}^{n}\|\alpha_{im} - \alpha_{ii}\| = K_0(0)$.
Then $\{\alpha_{im}\}$ is Cauchy sequence in real field $\mathbb{R}$ or complex field $\mathbb{C}$, since real field $\mathbb{R}$ or complex field $\mathbb{C}$ are complete , therefore ; for all I there exists $\alpha_i \in F$ such that $\lim_{n\to\infty} \alpha_{im} = \alpha_i \,, put\; x = \Sigma_{i=1}^{n}\alpha_i\, \eta_j$.
There exists $x \in A$ for all $A \in 2^{\Omega}$, $\lim_{n\to\infty} \; \inf_{\Sigma_{i=1}^{n}\alpha_{ii}\eta_j\in A} \; \delta_R\left(\Sigma_{i=1}^{n} a_{im}\eta_j, A\right) = K_0(0)$. Thus Ω is $\delta_R -$ complete .
This can be deduced from the fact that both $\mathbb{R}$ and $\mathbb{C}$ are complete .

Definition 3.6 : An a-random appr. normed space is called δ_R-complete if every δ_R −Cauchy sequence is δ_R-convergent in (Ω, δ_R).

Theorem 3.7 : An a-random appr. normed (Ω, δ_R) is δ_R-complete space if and only if (Ω, d_{δ_R}) is complete.

Proof: Let $\{ x_n\}_{n=1}^{\infty}$ be a Cauchy sequence in (Ω, δ_R) , then it is δ_R- Cauchy sequence in (Ω, δ_R) since (Ω, δ_R) is complete, there exists $x \in B$ for all $B \in \Psi(B)$,such that $\delta_R(\{x_n\}, B) = K_0(0)$,$\Psi(B)$ the set of all cluster point in a-random appr. space.
$\sup_{M\,\epsilon\,\Gamma(X)} \; \inf_{\substack{x\in M \\ x_{i\in A_i}}} \; d_\beta(\{ x_n\},\{x\}) = K_0(0)$ then $d_{\delta_R}(x_n \,, x) = 0$. That is (Ω, d_{δ_R}) is complete.
Conversely, Let $\{ x_n\}_{n=1}^{\infty}$ be δ_R- Cauchy sequence in $\left(\Omega, d_{\delta_R}\right)$. Hence, The sequence $\{x_n\}_{n=1}^{\infty}$ is left and right sequence in (Ω, d_{δ_R}).
(Ω, d_{δ_R}) is complete that is $\lim_{n\to\infty} \; d_{\delta_R}(x_n \,, x) = 0$
that is $\lim_{n\to\infty} \; \inf_{x\in B} \; \delta_R(\{a_n\}, B) = K_0(0)$ and $\lim_{n\to\infty} \; \sup_{x\in B} \; \delta_R(\{a_n\}, B) = K_0(0)$
$\delta_R(\{a_n\}, B) = \sup_{B\,\in\Psi\,(\Omega)} \; \inf_{x\in B} \; d_{\delta_R}(\{ A_n\},\{x\}) = K_0(0)$, that is there exists $x \in X$ and for all $B \in \Psi(\Omega)$, $\delta_R(\{a_n\}, B) = K_0(0)$
Hence $\{ x_n\}_{n=1}^{\infty}$ is convergent in a-random appr. space (Ω, δ_R).

Example 3.8 : Let $(\Omega, \|.\|_R)$be a Linear normed spaces . Define a mapping

$$\boldsymbol{\sigma}_x(t) = \begin{cases} 0 \;, if\; t \le 0 \\ \dfrac{t}{t + \|x\|} \quad , \quad if\; t > 0 \end{cases}$$

Then (Ω, δ_R) is a-random appr. normed space
Proof:
1) $\boldsymbol{\sigma}_x(t) = 1\; then,\; \frac{t}{t+\|x\|} = 1$ therefor, $\|x\| = 0$ hence , $x = 0$
the conversely , it is clear

2) $\boldsymbol{\sigma}_{\lambda x}(t) = \frac{t}{t+\|\lambda x\|} = \frac{t}{t+|\lambda|\|x\|} = \frac{t}{t+\|x\|} = \boldsymbol{\sigma}_x(t)$
3) $\lim_{\lambda\to 0} \boldsymbol{\sigma}_{\lambda x}(t) = \lim_{\lambda\to 0} \frac{t}{t+\|\lambda x\|} = \lim_{\lambda\to 0} \frac{t}{t+|\lambda|\|x\|} = \frac{t}{t} = 1 \quad , t > 0 \implies \lim_{\lambda\to 0} \boldsymbol{\sigma}_{\lambda x}(t) = K_0(t)$
4) $T_p\left(\boldsymbol{\sigma}_x(t), \boldsymbol{\sigma}_l(s)\right) = \frac{t}{t+\|x\|} . \frac{s}{s+\|l\|} . = \frac{1}{1+\frac{\|x\|}{t}} . \frac{1}{1+\frac{\|l\|}{s}} \le \frac{1}{1+\frac{\|x\|+\|l\|}{t+s}} = \frac{t+s}{t+s+\|x+l\|} = \boldsymbol{\sigma}_{w+l}(t+s)$
now ,show $(RN5)$

$$\delta_R(x,N)=\begin{cases} K_0(0) & ,if\ t\le 0 \\ sup_{x\in\Omega} inf_{a\in N}\dfrac{t}{t+\|x-a\|} & ,\quad if\ t>0\end{cases}$$

Now to prove $\delta_R(x,N)$ is a-random approach

1. $if\ t>0\ , \delta_R(x,\{x\}) = sup_{x\in\Omega} inf_{a\in\{x\}}\frac{t}{t+\|x-x\|} = 1\ , then\ \delta_R(x,\{x\}) = K_0(r)$.
2. $if\ \ N=\emptyset\ , \delta_R(x,\emptyset) = \ sup_{x\in\Omega} inf_{a\in\emptyset}\frac{t}{t+\|x-a\|} = K_\infty(r)$.
3. $Let\ N,B\in 2^\Omega, \delta_R(x,N\cup B) = \ sup_{x\in\Omega} inf_{a\in N\cup B}\frac{t}{t+\|x-a\|} =$

$min\left\{sup_{x\in\Omega} inf_{a\in N}\frac{t}{t+\|x-a\|}, sup_{x\in\Omega} inf_{a\in B}\frac{t}{t+\|x-a\|}\right\} = \min\big(\delta_R(x,N),\delta_R(x,B)\big)$.

4. $\delta_R(x,N)\ge\delta_R\big(x,N^{h(r)}\big)+h(t)\quad ,for\ any\ h(r)\in\nabla^+$.

4. New Results of δ_R-Contractions on a-random approach normed spaces

Proposition (4.1): If Ω_1 and Ω_2 are a-random appr . normed vector space, and $\varphi:\ \Omega_1\ \to\ \Omega_2$ is surjective linear function, Then the qualities listed below are equivalent:

1) φ: $\left(\Omega_1\,,\delta_{R_1}\right)\to\left(\Omega_2\,,\delta_{R_2}\right)$ is δ_R- contraction .

2) $\left(\Omega_2\,,\delta_{R_2}\right)$ is $\delta_{R_2}-$ complete space whenever $\left(\Omega_1\,,\delta_{R_1}\right)$ is $\delta_{R_1}-$ complete .

Proof:

1) If $\varphi:\ \Omega_1\ \to\ \Omega_2$ is δ_R- contraction . Then for every $x\ \in\ \Omega_1$ and each subset $M\subset\ \Omega_1$

$\delta_{R_2}\big(\varphi\ (A),f\ (M)\big)\ \ge\ \delta_{R_1}\ (A,M)\ if\ \left(\Omega_1\,,\delta_{R_1}\right)$ is a-random appr. Banach space.

To prove $\left(\Omega_2\,,\delta_{R_2}\right)$ is $\delta_{R_2}-$ complete space.

Let $\{y_n\}\ be\ a\ \delta_{R_2}$ – caushy sequence in Ω_2 then there exists $\{x_n\}$ such that $\varphi(\{x_n\})=\{\ y_n\}$

$\lim\limits_{n\to\infty}\ \inf\limits_{x_m\in M}\ \delta_{R_2}(\{y_n\},M)=K_0(0)\ then\ \lim\limits_{n\to\infty}\ \inf\limits_{x_m\in N}\ ,\delta_{R_2}\left(\varphi\ (\{x_n\}),£(N)\right)=K_0(0)$,

where $(N)=M$.

Since φ is δ_R- contraction .

$0=\ \lim\limits_{n\to\infty}\inf\limits_{x_m\in N}\quad\delta_{R_2}\left(\varphi\ (\{x_n\}),\varphi(N)\right)>\lim\limits_{n\to\infty}\ \ \inf\limits_{x_m\in M}\delta_{R_1}(\{x_n\},M)$.

Hence, $\lim\limits_{n\to\infty}\ \ \inf\limits_{x_m\in M}\delta_{R_1}(\{x_n\},M)=K_0(0)$. That is $\{x_n\}$ is δ_R- caushy sequence in Ω_1,

Ω_2 is δ_R- complete app- space. There exists $\in N$, for all $N\subseteq\Omega_1$. Such that $\lim\limits_{n\to\infty}\ \ \inf\limits_{x\in M}\delta_{R_1}(\{x_n\},N)=K_0(0)$.

$\delta_{R_2}\left(\varphi\ (\{x_n\}),\varphi\ (N)\right)\le\delta_{R_1}(\{x_n\},N)$

$\lim\limits_{n\to\infty}\ \sup\limits_{x\in M}\ \delta_{R_1}(\{\ x_n\},M)\ =K_0(0)$ and $\lim\limits_{n\to\infty}\ \ \inf\limits_{x\in M}\delta_{R_1}(\{x_n\},M)=K_0(0)$

$\lim\limits_{n\to\infty}\sup\limits_{x\in M}\delta_{R_2}\left(\varphi\ (\{x_n\}),\varphi\ (M)\right)\le\lim\limits_{n\to\infty}\ \sup\limits_{x\in M}\ \delta_{R_1}(\{\ x_n\},M)\ =K_0(0)$

$\lim\limits_{n\to\infty}\ \ \inf\limits_{x\in M}\delta_{R_2}\left(\varphi\ (\{x_n\}),\varphi\ (M)\right)\ge\lim\limits_{n\to\infty}\ \ \inf\limits_{x\in M}\delta_{R_1}(\{x_n\},M)=K_0(0)$

$\lim\limits_{n\to\infty}\sup\limits_{x\in M}\delta_{R_2}\left(\varphi\ (\{x_n\}),\varphi\ (N)\right)\le K_0(0)$

$\lim\limits_{n\to\infty}\ \ \inf\limits_{x\in S_2}\delta_{R_2}\left(\varphi\ (\{x_n\}),\varphi\ (M)\right)=K_0(0)$

$\lim\limits_{n\to\infty}\sup\limits_{x\in S_2}\delta_{R_2}\left(\varphi\ (\{x_n\}),\varphi(M)\right)=K_0(0)$

Then (Ω_2, δ_{R_2}) is δ_{R_2} −complete space
Conversely, suppose φ is not δ_R −contraction
$\delta_{R_2}\left(\varphi(\{x_n\}),\varphi(N)\right) \geq \delta_{R_1}(\{x_n\},N)$. Let $\{x_n\}$ be a δ_R − convergent sequence in Ω_1
That is $\{x_n\}$ is δ_R − caushy sequence in Ω_1, $\{\varphi(\{x_n\})\}$ be δ_R − caushy sequence in Ω_2
The condition hold then there is $\{\varphi(\{x_n\})\}$ in Ω_2. There exists $y = \varphi(x) \in \varphi(N) = M \in 2^{\Omega_2}$. Such that $\delta_{R_2}\left(\varphi(\{x_n\}),\varphi(N)\right) = K_0(0)$. That is $\delta_{R_1}(\{x_n\},N) < K_0(0)$, this impossible.

Proposition 4.2 : An a-random appr. normed space $(\Omega, \delta_R, \|.\|_{\delta_R})$ is δ_R-complete if and only if a metric approach space (Ω, d_{δ_R}) is δ_R- complete.

Proof: Let Ω be a-random appr. normed space. and δ_R is generated by the $\|.\|_{\delta_R}$.
Let $\{a_n\}_{n=1}^{\infty}$ cauchy sequence in (Ω, δ_R). Then we have $d_{\delta_R}(\{a_m\},\{A_n\}) = 0$ for all $m,n \in Z^+$.This implies that $\delta_R(\{a_n\},M) = \sup_{a_n\in S}\inf_{A_m\in M} d_{\delta_R}(\{a_n\},\{A_m\}) = 0$.That is $\inf_{A_m\in M}\delta_R(\{A_n\},M) = K_0(0)$.Then $\{a_n\}_{n=1}^{\infty}$ is δ_R- cauchy sequence in $(\Omega, \delta_R, \|.\|_{\delta_R})$.
Since Ω is δ_R- complete, this implies that there exist $A \in M$ for all $M \in 2^{\Omega}$, $\delta_R(\{a_n\},M) = K_0(0)$ for all $n \in Z^+$, $d_{\delta_R}(\{x_n\},\{x\}) = \inf_{x\in M}\delta_R(\{x_n\},\{x\}) = K_0(0)$ that is $\{x_n\}$ converge to .
Conversely, suppose that (Ω, d_{δ_R}) is δ_R- complete, and Let $\{a_n\}_{n=1}^{\infty}$ is δ_R-Cauchy sequence in $(\Omega, \delta_R, \|.\|_{\delta_R})$, then$K_0(0) = \inf_{A_n\in S}\delta_R(\{a_n\},M)$

$$= \inf_{M\in 2^{\Omega}}\sup_{a_n\in\Omega}\inf_{a_m\in M}\boldsymbol{\sigma}_{a_n-a_m}(r)$$

$$= \inf_{M\in 2^{\Omega}}\sup_{a_n\in S}\inf_{a_m\in M} d_{\delta_R}(\{a_n\},\{a_m\})$$

$$d_{\delta_R}(\{A_n\},\{A_m\}) = \inf_{M\in 2^{\Omega}}\inf_{a_m\in M}\delta_R(\{a_n\},\{A_m\})$$

$$= \inf_{M\in 2^{\Omega}}\inf_{x\in M}\inf_{a_m\in M}\delta_R(\{a_n\},\{a_m\}) = K_0(0)$$

$d_{\delta_R}(\{A_n\},\{A_m\}) \to 0$ as $n \to \infty$.That is $\{a_n\}_{n=1}^{\infty}$ is δ_R- cauchy sequence in (Ω, d_{δ_R})
(Ω, d_{δ_R}) is δ_R- complete, therefore $\{a_n\}$ is converge sequence,
There exists $x \in \Omega$ such that $\lim_{n\to\infty}\{x_n\} = \{x\}$.
$d_{\delta_R}(\{x_n\},\{x\}) = \inf_{M\in 2^{\Omega}}\inf_{\substack{A_m\in M\\ x_{i\in A_i}}}\delta_R(\{x_n\},\{x\}) = K_0(0)$.There exists $x \in M$ for all $\in 2^{\Omega}$,
such that $\delta_R(\{x_n\},M) = \inf_{M\in 2^{\Omega}}\sup_{x_n\in X}\inf_{x\in M} d_{\delta_R}(\{x_n\},\{x\}) = 0$, hence $(\Omega, \delta_R, \|.\|_{\delta_R})$ is δ_R-complete.

corollary 4.3: A A-random appr. normed space is A-random appr. Banach space if and only if (Ω, d_{δ_R}) is Banach space.

Proof: As a result of Remark 3.3 .

Proposition 4.4: Let $(\Omega, \delta_R, \|.\|_{\delta_R})$ be a a-random appr. normed space then the following are equivalent:
(1)$(\Omega, \delta_R, \|.\|_{\delta_R})$ is an A-random appr. Banach space.
(2)(Ω, δ_R) is complete .
The proof is clear by corollary 4.3.

Proposition (4.5): Let $(\Omega, \delta_R, \|.\|_{\delta_R})$ be an A-random normed space . then we have:
(1)The function $\varphi: (x\,, y) \to x + y$ is $\delta_R -$ contraction.
(2)The function $\varphi : (\alpha\,, y) \to \alpha x$ is $\delta_R -$ contraction.

Proof:
(1)Let $\{(x_n\,, y_n)\}$ be a convergent sequence in Ω. There exists $x\,, y, \in \Omega\, for\, all\, M, N \in \Psi\,(\Omega)$(respectively), such that $\delta_R\,(\{x_n\}\,, M) = 0\,, \delta_R\,(\{y_n\}\,, N) = 0.$ Since $\delta_R(x_n, M) = \sup_{x\in\Omega}\, \inf_{a\in M} \boldsymbol{\sigma}_{x_n - a}(r)$

$$= \sup_{x\in X}\, \inf_{M\subset X} d_{\delta_R}(x_{n,}x) = 0$$

$$\delta_R(y_n, M) = \sup_{x\in\Omega}\, \inf_{b\in M} \boldsymbol{\sigma}_{y_n - b}(r)$$

$$= \sup_{y\in\Omega}\, \inf_{M\subset\Omega} d_{\delta_R}(y_{n,}y) = 0$$

$$\delta_R\,(\varphi(\{x_n\}\,, \{y_n\}), \varphi(M, N)) = \acute{\delta_R}\,\,(\{x_n + y_n\}, M + N)$$

$$= \sup_{x,y\in\Omega}\, \inf_{M,N\subset\Omega} \boldsymbol{\sigma}_{x_n + y_n - a - b}(r)$$

$$\leq \sup_{x,y\in\Omega}\, \inf_{M,N\subset\Omega} \|x_n - x\| + \sup_{x,y\in X}\, \inf_{M,N\subset X} \boldsymbol{\sigma}_{y_n - b}(r)$$

$$\leq \sup_{x,y\in\Omega}\, \inf_{M,N\subset\Omega} d_{\delta_R}(\{x_n + y_n\}, \{x + y\}) = 0.$$

Then φ is sequentialy contraction , and therefor $\varphi\ is\ \delta_R -$contraction .
(2) Let $\{(\alpha_n\,, x_n)\}$ be a convergent sequence in $F \times \Omega$, then let $x \in X\,, for\, all\, M \in \Psi\,(\Omega)$. Such that $\delta_R\,(\{x_n\}, M\,) = 0$, $\delta_R\big(\varphi(\{x_n\}), f(M)\big) = \delta_R\,(\alpha\{x_n\}, \alpha M)$

$$= \sup_{x\in X}\, \inf_{M\subset X} \boldsymbol{\sigma}_{\alpha x_n - \alpha a}(r)$$

$$= \sup_{x\in X}\, \inf_{M\subset X} \boldsymbol{\sigma}_{\alpha x_n - \alpha x_n + \alpha x_n - \alpha a}(r) = K_0(0)\,.$$

Thus $\varphi\,(\{\alpha, x\}) = \{\alpha x\}$ is sequentialy $\delta_R -$contraction

Remark 4.6: Let $M = (\Omega, d_{\delta_R})$ a-random appr. metric space , then M is a Hausdorff space .

Proof: $Let\ a, b \in \Omega : a \neq b.$
Then from distinct points in a-random appr. metric space have disjoint open Balls exists open $\in -$ balls $D_\in\,(a)$and $D_\in\,(b)$ which contain, respectively, a and b in disjoint open sets.Hence the result by the definition of Hausdorff space.

Theorem 4.7: Every uniform a-random appr. normed space $(\Omega, \delta_R, \|.\|_{\delta_R})$ is a Hausdorff space.

Proof: Suppose that Ω^* be atopological duall of Ω . That is
$\Omega^* = \{\ : (\,\Omega, T_{d_{\delta_R}}\,) \to (\,R\,, T_S\,) \mid \varphi$ is linear and continuous functionals }.
Let T_Ω^* is the set of all non-negative closed unit ball in Ω^*, so $T_\Omega^* = \{\varphi \in \Omega^* : \varphi(x) \leq 1\}$ and the norm on duall is defined by
$\|\varphi\|_* = \inf_{x\subset T_\Omega^*} \|\varphi(x)\|$.
It is clear that $(\Omega^*\,, \|\varphi\|_*)$ is Banach space.
The duall of $(\Omega^*\,, \|\varphi\|_*)$ is called biduall of X which is denoted by Ω^{**}.
Let φ be non- empty subset of Ω^* the functional $\|x\|_\varphi: \Omega \to \mathbb{R}$ as followes:

$\|x\| = \sup_{£\in\varphi} |\varphi(x)|$ is a semi norm on Ω. We have $M_{\Omega^*} =\{ \|x\|_{\varphi}: \varphi \subset T^*_\Omega \}$ and $N_{\Omega^*}=\{d_{\|x\|_{\varphi}}: \varphi \subset T^*_\Omega\}$. Then a basis for the weak topology $\xi(\Omega, \Omega^*)$ on Ω is given by :
$\{\{$ b∈ X: for all $f \in \varphi$: $|f(x-b)| < \varepsilon : \emptyset \neq \varphi \subset \Omega^*, \varepsilon > 0\}$ for $x \in \Omega\}$. Define $\delta_{R_{\Omega^*}} : \Omega \times 2^\Omega \to \nabla^+$
by
$\delta_{R_{\Omega^*}}(x,N) = \sup_{\varphi \subset T_{\Omega^*}} \inf_{a\in N} \boldsymbol{\sigma}_{x-a}(r)$.
It is clear $\delta_{R_{\Omega^*}}$ satisfies the conditions of approach distance, is said to be weak distance or weak approach distance. Since $\delta_{R_{\Omega^*}}$ is the uniform a-random appr. normed space generated by N_{Ω^*} ,An app-basis for the T^*_Ω is $M_{X^*} =\{ \|x\|_{\varphi}: \varphi \subset T^*_\Omega \}$ equall a basis for a weak topology $\xi(\Omega, \Omega^*)$ is given as:
$\{\{$ b∈ X: for all $f \in \varphi$: $|f(x-b)| < \varepsilon : \emptyset \neq \varphi \subset \Omega^*, \varepsilon > 0\}$ for $x \in \Omega\}$ that is equally a basis for weak topology $\xi(\Omega, \Omega^*$ is Hausdorff, then the a-random appr. normed space is Hausdorff space.

5.Conclusion

In this paper we study the convergent sequence in a-random approach space and sequentially convergent are discussed and the cluster point, open and closed ball and linear transformation. We are going to explain a a-random approach normed space . Every a-random approach normed space is a-random normed space, an a-random approach normed (Ω, δ_R) is δ_R-complete space if and only if (Ω, d_{δ_R}) is complete. A-random approach normed space is a-random approach Banach space if and only if (Ω, d_{δ_R}) is Banach space. Ω Every uniform a-random appr. normed space $(\Omega, \delta_R, \|.\|_{\delta_R})$ is a Hausdorff space.

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